

TIME HISTORY SIMULATION IN SEISMIC DESIGN

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Abstract: In seismic design a variety of methods can be used in order to assess the response of the structure. These methods are discussed with respect to their impact on the accuracy of the results and the possibility to verify the findings. When using harmonic base excitation usually the natural frequency of the structure is used as driving frequency. This might lead to unsafe results in case when damping or nonlinearities such as plastic actions of the structure are present. This paper focuses on a study on the appropriate driving frequency.

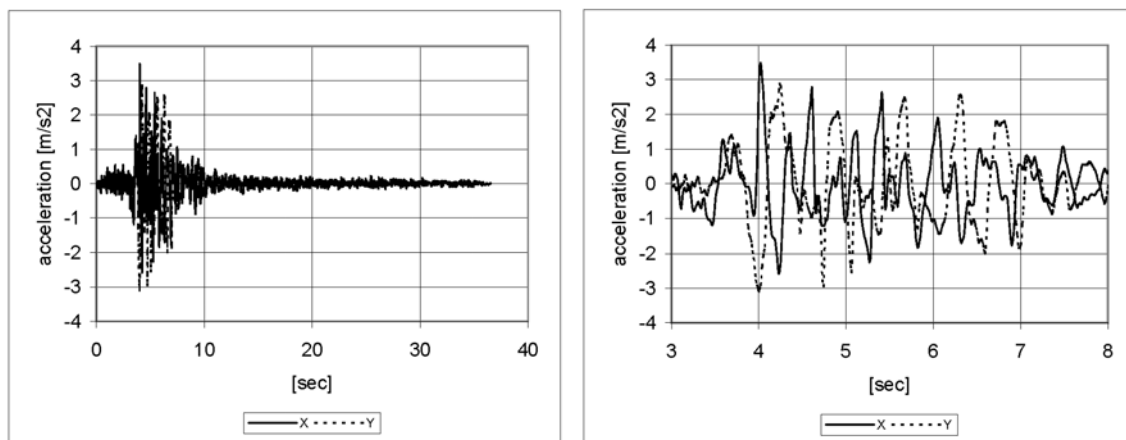


Fig. 1: Recorded horizontal accelerations during the Tolmezzo/Friuli (Italy) earthquake 06.05.1976 [1]

1 Introduction

In seismic design the most accurate but most laborious way is to model the structure and use recorded or artificial seismic data for the excitation of the structure. Implications of this type of modelling are discussed in [2]. Rules for this procedure are given in EC8-1 3.2.3.1 [3]. Alternatively a more simple procedure can be used where e.g. 10 sinusoidal waves are used as base excitation [4, 5]. In this case the chosen driving frequency of the forced motion should be such that maximum response of the structure is found. Generally with elastic structures this is coincident with the natural frequency of the structure.

At a closer look this is more complicated, because structures are detuned if damping is present. Furthermore, when nonlinearities are present, which is the case with plastic structures, other effects like foldover of the amplification function might be relevant.

In the present study we investigate a single storey frame. We try to answer the question which frequency should be used to obtain maximum storey-drift.

2 Terms and Definitions

ζ or D	damping ratio
δ	logarithmic decrement
$\eta = \Omega/\omega$	normalised driving frequency Ω with respect to the natural frequency ω
η (in EC8)	damping correction factor (eq. 3.6)
A	amplification, see DAF
DAF	dynamic amplification factor;
	$DAF = w_{dynamic} / w_{static} = w_{response} / w_{drive}$ (1)
	if a range of frequencies is under consideration, factor turns to function
SDOF	single degree of freedom
MDOF	multiple degrees of freedom
w	characteristic displacement, e.g. storey drift

3 Seismic Design

When designing a structure under seismic impact, a variety of methods and procedures can be used, which differ in the tools needed, the required degree of abstraction from the actual situation, the effort needed, the difficulty to verify the results, and finally, the accuracy of the outcome, see Table 1.

Remarks and explanations to the statements in Table 1:

- (A) compared to the response of the real structure under real seismic loads
- (B) according to EC8-1 4.3.3.2 [3]
- (C) the structure is regarded as SDOF oscillator; this implies with multi-storey buildings that higher modes will not be contributing significantly to the base shear
- (D) according to EC8-1 3.2.2.5 eqs 3.13-3.16 [3] a damping ratio of $\zeta = 5\%$ corresponding to a logarithmic decrement of

$$\delta = 2\pi\zeta = 2\pi * 0,05 = 0,314 \quad (2)$$

- is implied. This results from the fact that the damping correction factor η is not used with the design spectrum, even if one goes for $q = 1$. With steel chimneys it requires a medium size damping facility to achieve this damping ratio. This might be another inconsistency in EC8 [3]. For further discussion we refer to [2].
- (E) from another point of view this could be regarded as low, since the given response spectrum is postulated to cover all relevant seismic events
- (F) “low” is referring to the prediction of the response in an individual seismic event; at the same time this result is highly conservative
- (G) using formulae from mechanical engineering, such as “machine and foundation”
- (H) the formulae (G) are only defined for linear oscillators
- (J) the assumption of a harmonic drive (forced oscillation) is far from reality, see Fig. 1
- (K) multiple degrees of freedom are possible
- (L) the response of the model can be verified in the elastic range by means of (G) in the plastic range only plausibility checks are possible
- (M) this method was used in a previous paper of the authors [4]

- (N) recorded (see Fig. 1) or artificial seismic time-acceleration or time-displacement data are used for the base drive; the effort is extreme from a practical designers point of view; within academic environment the effort is regarded “normal [6]
- (O) *a priori* the results can not be verified; plausibility checks require the model to be simplified, thus stepping back towards (G)

Table 1: Tools and methods in seismic design

Tool	Method	Keyword	Abstraction in Structure	Abstraction in Loads	Effort	Verification	Accuracy (A)
hand	static	lateral force method (B)	SDOF (C) implicit plastic (D)	high (E)	low	by hand	low (F)
hand	dyn	steady state (G)	SDOF elastic (H)	high (J)	low	by hand	low
num	dyn	harmonic drive	MDOF (K) plastic (L)	high	medium	plausibility	middle
num	dyn	restricted harmonic (M)	MDOF plastic	high	medium	plausibility	improved middle
num	dyn	transient	as desired	none (N)	extreme	very difficult (O)	high

From the above compilation in Table 1 it can be seen that high accuracy in the results can only be achieved if extreme effort is spent on modelling and an proper performance of the time history calculations.

Hand calculations on the other hand are restricted to the lateral force method, which is a quasi-static approach. The required information on the dynamic amplification is given by the factor 2,5 in EC8-1 3.2.2.2 eqs 3.2-3.5 or 3.2.2.5 eqs 3.13-3.16 [3]. This factor has been derived by a series of numerical simulations (e.g. described in [7]), which have been performed – from the designers point of view – in the background of a code committee.

Interpreting the factor 2,5 as dynamic amplification of a SDOF oscillator (see eq. 5) this value is associated with a logarithmic decrement

$$\delta = \pi / A = \pi / 2,5 = 1,26 \quad (3)$$

or, in terms of damping ratio

$$\zeta = 1 / 2A = 1 / (2 * 2,5) = 0,20 \quad (4)$$

It is obvious that this does not describe the damping properties of a steel structure under steady state harmonic drive. It rather describes the fact that the drive is active for a certain number of cycles only and thus the response of the structure remains much smaller than it would develop in steady state condition. From Fig. 1 it can be seen that a real seismic event has only some 5 to 7 major peaks which are in the magnitude of the maximum acceleration. Of course, this might not be a general feature for all possible seismic events. In a previous paper the authors suggested that a drive with 10 periods should be sufficient conservative [4]. EC8-1 3.2.3.1.2 (3) [3] demands the drive to be active for 10 seconds. This seems to be very restrictive for structures with a natural frequency well below 1 Hz.

The above discussion shows that a dynamic hand calculation for steady state harmonic driven systems, as given in Table 1 row 2, will lead to very conservative results inevitably. In a real seismic event the structure has by far not enough time to adapt their response to steady state. The only advantage of this method is that there are analytic solutions which can be applied easily.

In order to avoid over conservative results, the designer should go for numerical methods which allow transient time-history analyses. On one hand this allows to incorporate material nonlinearities such as plastic material properties. On the other hand the results cannot be verified in a straight forward manner. Only plausibility checks are possible, which might involve very arbitrary decisions of the designer. The quality of the model with respect to mesh size and length of incremental time steps can only be checked by “downsizing” the model back to linear elastic properties and then calibrating against the steady state calculations described previously.

4 Basics of Nonlinear Vibrations

The dynamic amplification function is depicting the relation of the response amplitude of an oscillator at steady state and the amplitude by which it is driven, see Fig. 2. Linear oscillators are known to have a pole at resonance if undamped and a finite amplification given by

$$A = \pi / \delta = 1 / 2\zeta \quad (5)$$

According to EC1-1-4 Annex F.5 Tab. F.2 [8] steel structures such as swaying steel chimneys might have logarithmic decrement as small as

$$\delta = 0,015 \quad (6)$$

corresponding to a damping ratio of

$$\zeta = \delta / 2\pi = 0,015 / 2\pi = 0,00239 \quad (7)$$

which results in an amplification factor as large as

$$A = \pi / 0,015 = 1 / 0,00478 = 209 \quad (8)$$

Mark that the resonance peak of the amplification function is very small, see Fig. 2. Within an interval of only $\pm 5\%$ the amplification changes from 10 to close to infinity and back to 10.

When loading and displacement are not proportional, structures are being referred to as being nonlinear. This holds for most steel structures and is caused by second order effects, slip in bolted joints, contact effects between adjacent parts or with the foundation, plasticity at hot spots. All these effects are considered to be small and are considered to be negligible in design without causing too much error. Considerable nonlinearities which should not be neglected in design would be guyed masts or chimneys, where the stiffness increases with increasing displacement [9] and X-braced structures with tension diagonals [10] or elastic-plastic structures, where the stiffness decreases with increasing displacement [4].

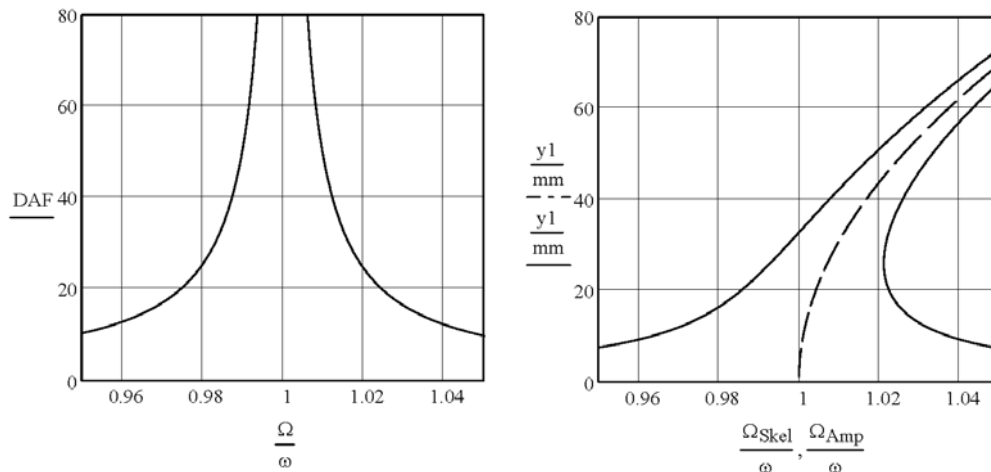


Fig. 2: Amplification graph for undamped oscillator at resonance
 a) linear oscillator with pole
 b) nonlinear example-oscillator (Duffing type [11]) with hardening stiffness showing foldover and backbone curve [9]

From an academic point of view, nonlinear structures have no natural frequency, because

$$\omega = \sqrt{c/m} \quad (9)$$

is not defined uniquely when the stiffness c is not a constant. From an engineering point of view it would be highly unsatisfactory to have no natural frequency. So, we still speak of natural frequency, referring e.g. to the inverse of the peak to peak distance, because we still have a periodic motion, although not a harmonic one. In special cases a quasi-constant stiffness is developed for a certain deformed state, e.g. with guyed masts. It is then possible to perform linear dynamic calculations for a nonlinear deformed state [12].

If the nonlinearity of the structure can be described by a third order function, the term “Duffing-oscillator” is used [11]. Straight forward analytical solutions allow to plot the response amplitude and the so called backbone curve, see Fig. 2 and Fig. 3. A recent, very elaborate treatise is given in [13]. Looking at the right hand side of the graph in Fig. 2, we see that for amplitudes bigger than resonance with $\eta = \Omega/\omega = 1$ it is possible that for a certain driving frequency we might have two or three associated response amplitudes. With static problems like buckling of imperfect structures we can assume that the structure will go for the minimum internal work. With dynamic structures like guitar strings we know that they can follow higher modes when excited adequately. In the transition zone there can be irregular motions, noted as jump phenomenon [14], see Fig. 3. With elastic plastic structures we should assume that the degree of nonlinearity is rather high, so that there might be a pronounced jump phenomenon.

Besides discovering jump phenomena the basic question is which driving frequency should be used in a numerical simulation in order to get maximum response of the structure. For linear structures the detuning is given by

$$\eta = \sqrt{1 - 2\zeta^2} \quad (10)$$

which describes the reduction of natural frequency in presence of damping. For low damped steel structures this shifting can be neglected,

$$\eta = \sqrt{1 - 2(0,00239)^2} = 0,999994 \quad (11)$$

even with 5 % damping the shift is negligible.

$$\eta = \sqrt{1 - 2(0,05)^2} = 0,997 \quad (12)$$

Since plastic actions of a structure cause reductions in amplitude which are by far higher than 5 % damping it should be assumed that detuning is not negligible. It should be pointed out

that the graphs in Fig. 2 and Fig. 3 with a foldover to the right represent oscillators with hardening stiffness. With elastic-plastic structures, having an overall softening stiffness, we should expect the foldover leaning to the left.

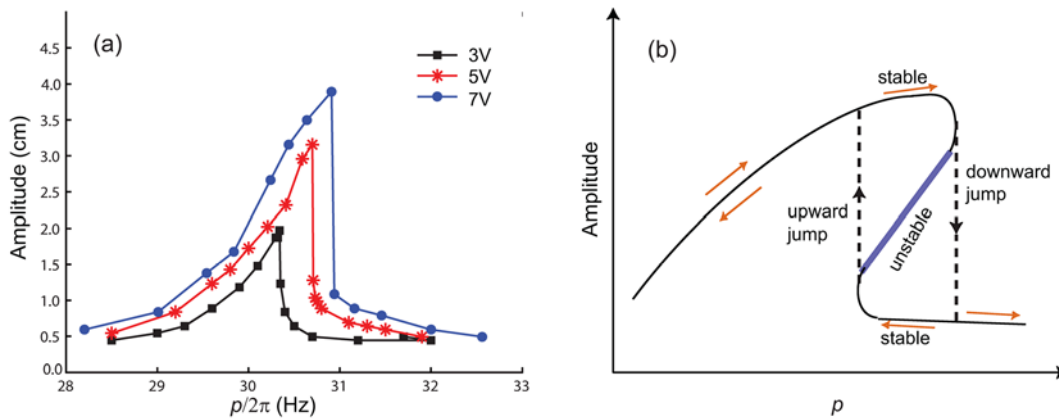


Fig. 3: Jump phenomenon [14]
 a) experimental data from a vibrating string
 b) modified response curve with unstable region

5 Numerical Study

We performed a numerical study using a pin-based plane frame of 6 m width and 4 m height. Tubular members 240x12 and 1000x12 are used for columns and waler. Steel with $f_y = 235$ MPa is used. After yielding the slope of the bilinear constitutive law is reduced to 625 MPa. The driving amplitude was chosen to be 5 mm at the column bases. The other data are corresponding to those given and discussed in [4, 5], background information is given in [15].

On the left hand graph of Fig. 4 a comparison is given of the amplification function regarding the (absolute) displacements of the waler (storey-drift) and the differential (relative) displacements between the driven column bases and the waler.

a) absolute displacements

$$\text{DAF} = 1 \quad \text{at} \quad \eta = 0$$

very slow drive; the waler is following the column bases without difference

$$\text{DAF} = 0 \quad \text{at} \quad \eta = \infty$$

very quick drive; the waler cannot follow the column bases but is at rest, the bases oscillate under the waler

b) relative displacements

$$\text{DAF} = 0 \quad \text{at} \quad \eta = 0$$

very slow drive; no difference between waler and bases

$$\text{DAF} = 1 \quad \text{at} \quad \eta = \infty$$

very quick drive; the difference equals the amplitude of the bases

With respect to stress resultants which cause plastic strains in the corners the differential displacements would be governing. With respect to plotting the numerical results the absolute response is easier to plot. We choose the second option; we can see in Fig. 4 that around $\eta = 1$ there is only little difference between both.

The response of the structure (storey drift) was regarded as amplitude of a motion. The biggest absolute value was recorded irrespective of the sign.

Elastic results are marked with hollow squares. It can be seen that we did not succeed in simulating a target amplification of 209 but rather an amplification of not more than 93. The maximum occurs at $\eta = 0,997$ which is not in accordance with the prediction.

Remark: the presence of a damping ratio of $\zeta = 0,00239$ has been verified by means of a die out simulation.

Interesting, but not investigated any further in this paper: the numerical model yields considerable higher amplifications than theory would predict.

Plastic results are marked with filled squares, see Fig. 4 and Fig. 5. Mark that the elastic limit storey drift is app. 52 mm, so that plastic action would begin for amplifications of the storey drift bigger than 10. It shows that for $\eta = 0,9$ and $1,1$ the mismatch of the driving frequency is big enough to limit the amplification to 9,2. Maximum amplifications of 16,0 were reached for $\eta = 1,003$. There is no marked peak in the amplification as the drive passes the resonance.

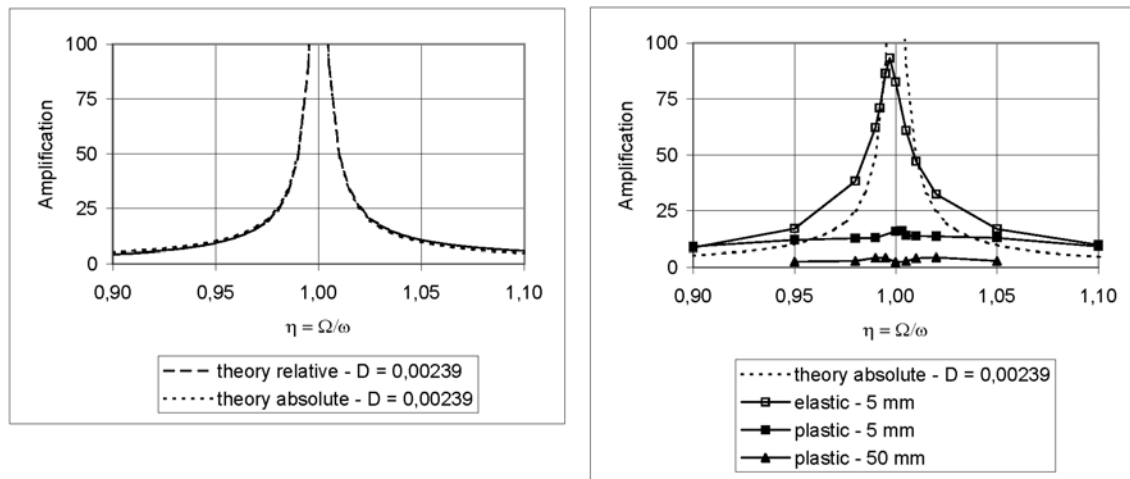


Fig. 4: Dynamic Amplification

- a) Theory – comparison of absolute and relative displacements
- b) Results of present numerical study

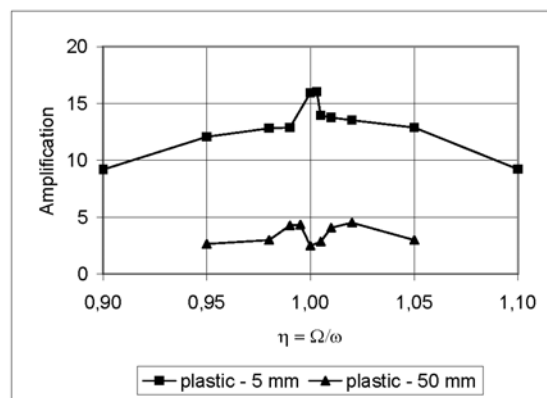


Fig. 5: Dynamic Amplification – plastic results of present numerical study

A second series of plastic simulations was run with an driving amplitude of 50 mm, the results of which are plotted in filled triangles, see Fig. 4 and Fig. 5. With this drive plastic strains occur when the amplification exceeds 1, which usually occurs in the first peak already. Maximum amplifications of 4,5 were reached for $\eta = 1,02$.

Mark that the behaviour factor according to the definition in EC8-1 3.2.2.5 (3) [3] or eq. (26) in [4] respectively is $93/16 = 5,8$ for 5 mm drive.

6 Verification

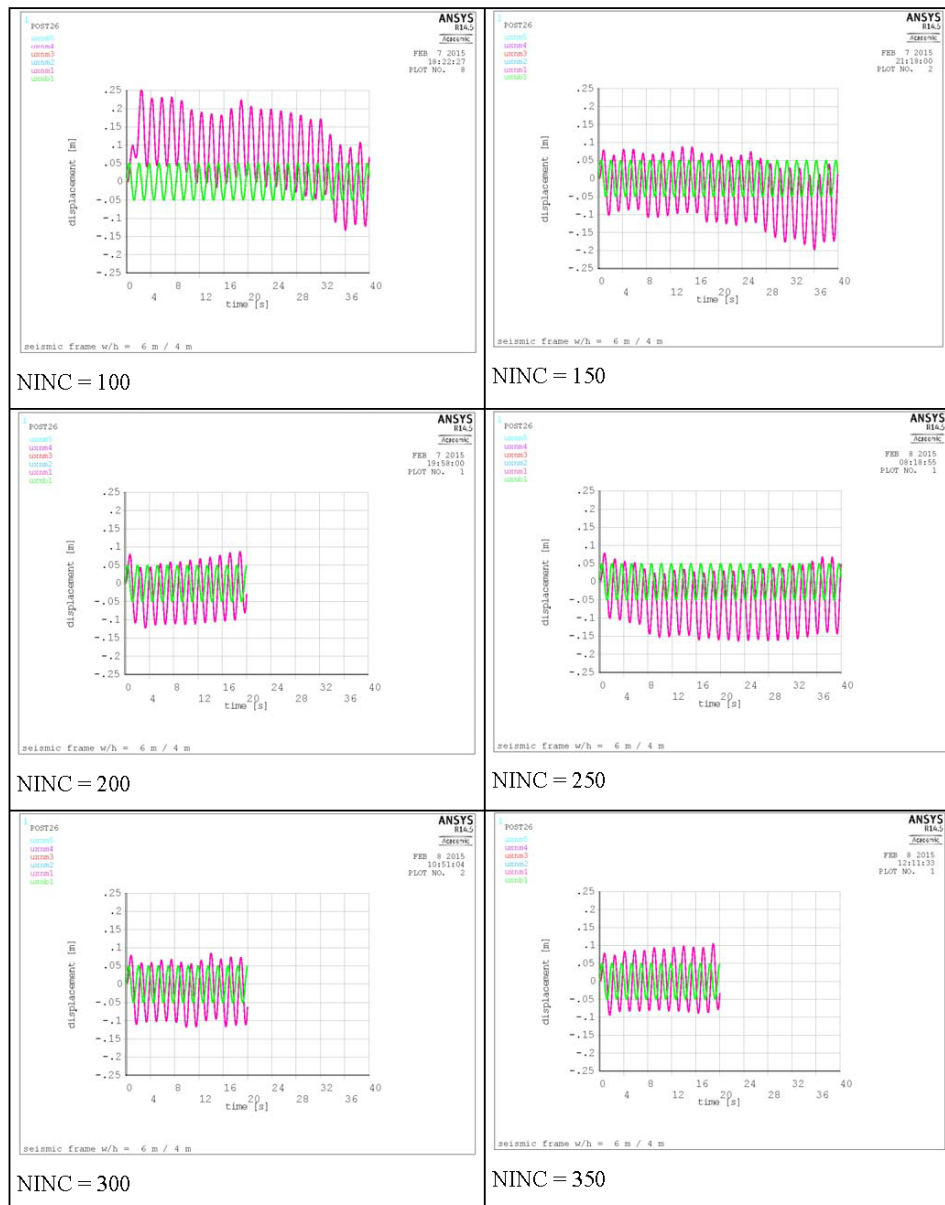


Fig. 6: Numerical results – shapes of load-displacement curves; sinusoidal drive with 50 mm amplitude and response of the structure; only the parameter NINC has been varied; previously, a value of NINC = 20 has been decided to be sufficient;

As mentioned while commenting on Table 1 the results of plastic runs can be verified only by plausibility checks due to the lacking of objective criteria. In FEA usually two hypotheses are used which hold for most applications:

- a) meshing – smaller element sizes improve results

A mesh is tested by subsequently reducing the element size (and increasing the number of elements as well as I/O-time) to the point where characteristic results such as hot spot stresses seem to approach to a limit value. Then usually a decision is made on how much error would be acceptable with respect to the target results. This might be

e.g. 5 % or 10 % of the “real” value. Based on this decision the element size is increased again to reduce I/O-time.

- b) path tracing – smaller load step increments improve results
Large load steps cause the solution to drift off the convex load-displacement-curve of elastic-plastic steel structures. Thus the load steps need to be sufficiently small to keep the solution tight to the “real” curve. Testing method and final decision is as given for the mesh size under clause a).

The model used for the present study was tested in the way described on the basis of an assumed 1 % error limit. The values received and the decisions made are documented in [15]. Based on this procedure it was decided that a beam element length of 200 mm was sufficient. The driving sinusoidal wave was divided by use of the parameter NINC, so that the length of the time step is given by the length of a double-half-wave divided by NINC. Additionally, within a time step a minimum of 10 iterative substeps were used. Based on an elastic die out simulation where the response of the structure has been checked along 11 periods it had been decided that $NINC = 20$ would be a sufficient size of time steps. Reducing the time steps to $NINC = 50$ did not change the response period by more than 1 %.

According to the above description of the verification procedure, reducing the time steps by values of $NINC > 50$ should give no significant change in the results. However, as can be seen from Fig. 6 even the response curves of the runs with $NINC = 300$ and $NINC = 350$ can be distinguished. As a compromise we decided that $NINC = 200$ should be sufficient for the purpose of this study. With a driving amplitude of 50 mm a division of 1000 substeps was not sufficient to have convergence in the first peaks, a parameter of 10.000 was used instead.

The feature of a drifting mean displacement under nearly constant amplitudes is known with plastic structures, it has been studied in [16].

7 Conclusions

Following previous studies on the seismic behaviour of elastic-plastic steel structures numerical studies have been performed which focus on the methods and required accuracy of these calculations.

The main conclusions are:

1. Plastic steel structures do not exhibit a pronounced resonance peak as known with elastic steel structures.
2. Still, missing the resonance frequency by 1 % might give results which are by 20 % unsafe.
3. A Duffing-type jump phenomenon could not be observed.
4. No advice on detuning can be given since we found variations to both sides of the elastic natural frequency. Thus we recommend to have several runs close to the resonance in order to check the sensitivity of the system and to find the peak of the amplification function.
5. “Ordinary” engineering decisions on how much accuracy is needed with respect to the length of the period might be wrong for this type of problem. It is evident that limiting the error of the period to 1 % is not a sensible measure if the response of the structure varies by 20 % around ± 1 % of the resonance.
6. It is underlined again by this findings that FE analyses need a very thorough documentation which includes the verification procedure. Otherwise the results should not be considered trustworthy.

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