‘Clean’ Sinusoidal Response vs. Speed in Fatigue Testing

P. Knoedel[1], F. Steidl[2], Th. Ummenhofer[3]

[1]Dr Knoedel Engineering Consultants, Vordersteig 52, D-76275 Ettlingen;
info@peterknoedel.de;
[2,3]KIT Steel & Lightweight Structures, Karlsruhe Institute of Technology, D-767131 Karlsruhe;
frank.steidl@kit.edu;

Abstract

When tuning the testing machine to perform a constant-amplitude fatigue test, a sinusoidal input signal is used in most cases. The actual response of the dynamic system of machine and specimen is checked by plotting the output signal with an oscilloscope. The staff in charge of the machine are trying to achieve the highest possible frequency in order to keep testing short. On the other hand the load output signal of the system might show higher harmonics which are superimposed to the lowest sinusoidal wave. Although no such order is given the staff then tends to lower the frequency until they arrive at a ‘clean’ sinusoidal wave shape again.

This seems to be a very conservative approach, if the output signal is interpreted by means of Fourier series. A higher harmonic with a smaller amplitude being present results in a load spectrum with a big-amplitude-part, being the desired testing load, and a small-amplitude-part which is being tried to be avoided by the staff. It is generally acknowledged however that if the amplitudes are small enough they do not contribute to the Miner’s sum of accumulative damage.

In the present paper we show some robust rules for higher speed in testing – discarding the ‘clean’ sinusoidal response.

1. Introduction

This is a testing lab paper, it addresses issues how to operate fatigue testing machines.

When installing a fatigue test setup and tuning the testing machine the target limit loads are of course the main parameters. Second important is the speed, i.e. the frequency, with which the loads will be altered during the test.

According to the experience of the authors it is common practice to speed up the machine as much as possible and watch the response signal on the screen. When the sinusoidal wave shape of the response signal begins to show noticeable aberrations from a ‘clean’ sine, such as denting, fluttering, global triangular shape, a plateau at load peak, etc., the operator will reduce the frequency until a sinusoidal shape is achieved to a desired degree of beauty. It seems, that this is an overall behaviour pattern of fatigue test operators, although nobody ever heard of a working instruction – written or spoken –, in which an order of that kind was given.

In this study we will investigate the background of this myth and try to achieve higher testing speeds.

2. Constant Amplitude Fatigue Test

In many cases fatigue tests are performed as constant amplitude test, sometimes referred to as Wöhler-Test. Stress amplitude and mean stress remain constant over all testing cycles. In terms of block loading (amplitudes remain constant within each block) this is referred to as ‘single-block test’ and in terms of spectrum loading it is a single-peak spectrum.

In fatigue design it is commonly assumed that the shape of the load cycles in time is arbitrary, because we know from e.g. rainflow counting method [1], that only the variation of stress is significant, but not the shape of the load history. Due to practical reasons a sinusoidal load history is used in most cases. With e.g. servo-valve driven testing machines a target signal is input to the dynamic system and the actual response is output to a screen.

The staff operating the machine will arbitrarily increase the testing frequency up to a point, where the sinusoidal response shows noticeable deviations from a clean sine. Experience with a lot of testing labs shows that there are no working instructions which bind the staff to do so. It seems to be a kind of common understanding that the sinusoidal response should be clean. In the early 80s the first author was running fatigue tests on a servo hydraulic machine at KIT himself, so this is also very much based on own experience.

3. Non-Harmonic Response

If we increase the testing speed beyond the frequency where we have a clean sinusoidal response, we get noticeable differences from the sinusoidal shape, two examples of which are given in the graphs in fig. 1.

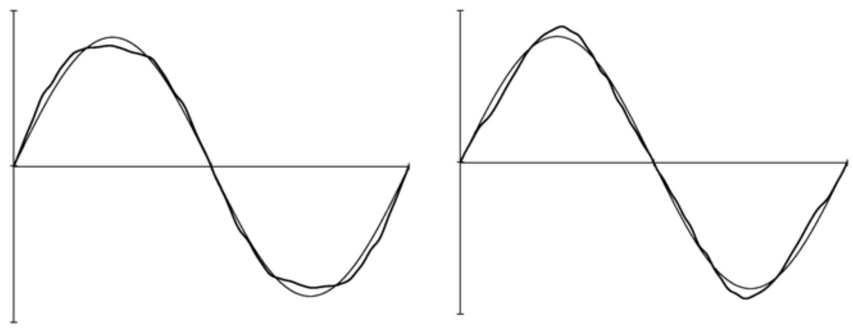

Figure 1: Degraded sinusoidal response in fatigue test,
represented by a 30-term Fourier sum
left: KIT, axial, F = 60 kN ± 10 kN, 14 Hz
right: LaborSL, bending, F = 12 kN ± 7 kN, 22 Hz

The data shown in fig. 1 originate from tests with dummy specimen during February 2012 at KIT Steel & Lightweight Structures, Karlsruhe, and at Labor für Stahl- und Leichtmetallbau GmbH at the University of Applied Sciences, Munich.

In both cases the staff would not have continued the test in that way. KIT would have reduced the frequency to 8 Hz, to receive a seemingly clean sine, LaborSL would have reduced to 17 Hz.

4. Interpretation by Fourier Decomposition

In trying to assess the effects of the differences to the intended sinusoidal shape we perform a harmonic analysis on the response time history. By using the notation of Bronstein [2] we have a truncated sum of Fourier series given by

$$s_r(x) = \frac{a_0}{2} + \sum_{k=1}^{r} a_k \sin\left(k\frac{2\pi x}{T}\right) \qquad \text{(eq. 1)}$$

where the number r of the summed terms needs to remain smaller than the number n of coordinate pairs which we take from the sample. Mark that the cosine terms which are generally included in the sum of Fourier series may be omitted if we neglect phase-shifting. This holds, if we restrict to sampling between pairs of zero crossings.

The Fourier coefficients a_k are determined as follows:
we divide the oscillation period in $2n$ equally spaced parts, the abscissae of the sections are given by

$$x_k = \frac{kT}{2n} \quad \text{(eq. 2)}$$

The associated ordinates $f(x_k) = y_k$ with $k = 0, 1, \ldots , 2n$ are read from the database of the testing machine or extracted from a digitized plot. Then the coefficients are given by

$$a_0 = \frac{1}{n}\sum_{k=0}^{2n-1} y_k \; ; \quad a_m = \frac{1}{n}\sum_{k=0}^{2n-1} y_k \sin\left(\frac{km\pi}{n}\right) \quad \text{(eq. 3)}$$

with $m = 1, 2, \ldots , n$.

For this study we decided 30 pairs of coordinates to be sufficient, the plots in fig. 1 represent the truncated sum of Fourier series according to eq. 1. Since the Fourier coefficients are identical with the amplitudes of the respective frequency, they represent an amplitude spectrum in the frequency domain, see fig. 2. Most of the higher harmonics have very small amplitudes, so we plotted logarithmic magnitudes which are normalized to the amplitude of the main harmonic $m = 1$.

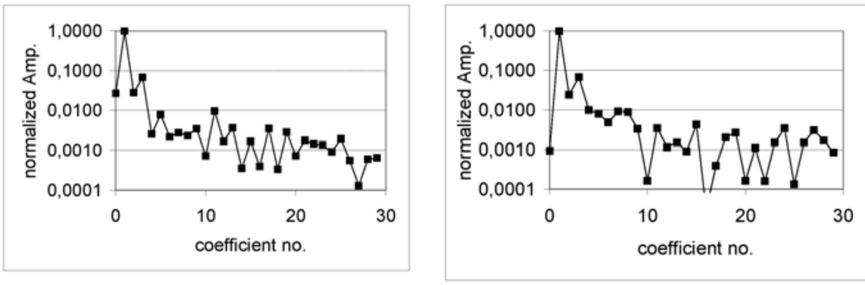

Figure 2: Normalized amplitude spectrum of the curves in fig. 1; logarithmic scale; left: KIT; right: LaborSL

We see that the amplitudes are decaying rapidly towards higher harmonics. From the fluttering lines in fig. 1 we would rather have expected an increase of the amplitudes at $m > 25$.

5. Cumulative Damage

Cumulative damage is assessed by using Miner's rule [5] as given in EC3-1-9 [1]. Although the shortcomings of the Palmgren-Miner hypothesis [6] are well known [3], the hypothesis is a valuable tool anyway which is widely used when assessing block or spectrum loading.

In this paper we assume that a Fourier representation of the stress history with block loading according to the single harmonics is equivalent to the original stress history itself. This is a very delicate assumption when it comes to wave-shapes like in fig. 1 left hand side, where the actual peak load is lower than the amplitude of harmonic $m = 1$. It implies that we have a damage – at least a virtual part in a damage scenario – which is not associated with an actual stress variation. As far as the authors know this assumption is not verified so far.

We introduced the normalized spectra given in fig. 2 into a EC3-Palmgren-Miner-spreadsheet. Arbitrarily we chose a scenario with an upper stress of 200 N/mm^2 at $R = -1$, giving $\Delta\sigma = 400$ N/mm^2 and 10.000 main cycles. Compared to detail category 80 we receive a cumulative damage of 0,625 from the main harmonic. The next-lower amplitude $\Delta\sigma = 27$ N/mm^2 is associated with the third harmonic; it is lower than the cut-off limit and thus does not contribute to damage. A graphical representation is given in fig. 3.

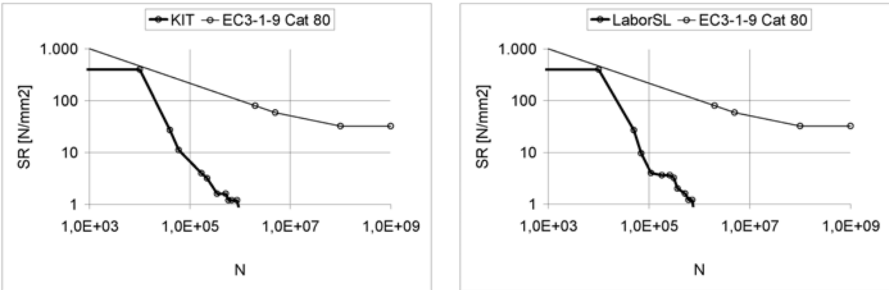

Figure 3: Most unfavourable scenario for a fatigue test with the amplitude spectra given in fig. 2; left: KIT; right: LaborSL

It can be taken from the above graphs that if the main harmonic had been reduced in stress level, yielding a higher number of cycles in the fatigue test, the higher harmonics would have fallen even deeper under the cut-off limit and thus being insignificant.

6. Increasing Testing Speed

From the above results we see that it would have been possible to conduct the tests at high speed without influencing the test results by higher harmonics.
KIT could have tested at 14 Hz instead of 8 Hz, reducing net testing time by 40 %;

LaborSL could have tested at 22 Hz instead of 17 Hz, reducing net testing time by 20 %.

How degraded does a sinusoidal shape look, if it does influence the test results? We chose 5 % error to be ‘small’ with respect to the scatter of fatigue results and were – by trial and error – determining two-block-spectra, where the second block should have an amplitude as to increase the cumulative damage of the first block by 5 %. We did this for several harmonics, the results are given in fig. 4.

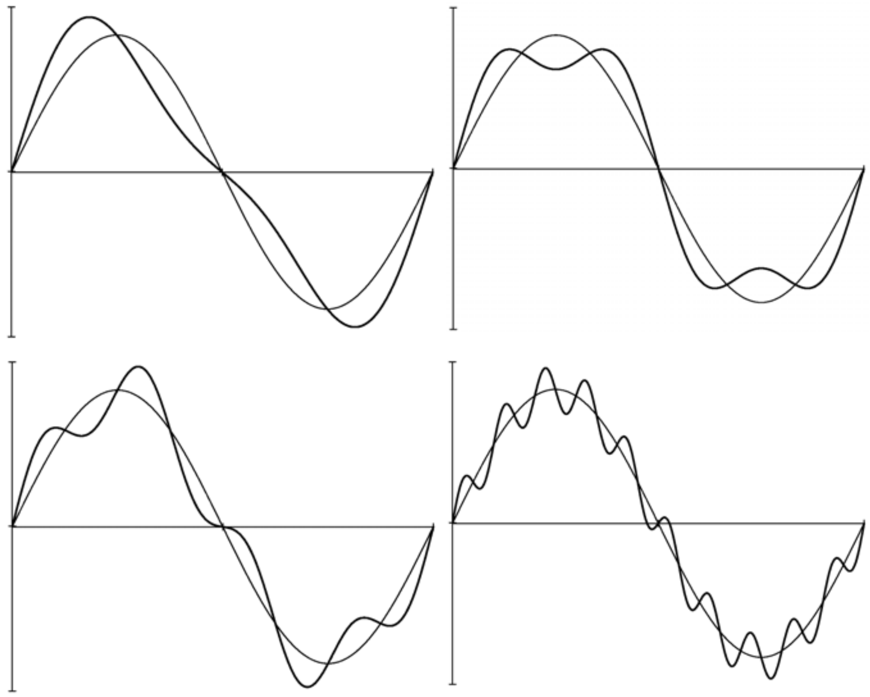

Figure 4: Degraded sinusoidal shapes which produce 5 % damage additional to the first harmonic; the normalized size of the second amplitude is given in brackets
top left: 1. + 2. harmonic (0,293); top right: 1. + 3. harmonic (0,256);
bottom left: 1. + 4. harmonic (0,232); bottom right 1. + 10. harmonic (0,171);

It can be taken from the graphs in fig. 4 that only gross aberrations of the basic sinusoidal shape produce an error as small as 5 %. These response shapes would surely not have been accepted by any lab staff.

7. Conclusions

In this study we had a closer look on testing speed vs. clean sinusoidal response of the test setup.

- What looked like a distinct aberration of the desired sinusoidal response of the test set up which did not seem to be acceptable to the staff operating the testing machine, turned out to have such small amplitudes in the higher harmonics that no contribution to the cumulative damage could be stated.
- This is due to the shape of the amplitude spectrum which shows a surprisingly strong decay after the first harmonic.
- The findings indicate that fatigue testing can be performed at significant higher frequencies than before, thus saving time and resources.
- Aberrations from the sinusoidal shape which do produce significant errors in the test results are really gross – examples are given.

8. Acknowledgements

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9. References

[1] EN 1993 Eurocode 3 (EC3): Design of steel structures. Part 1–9:2005 + AC:2009: Fatigue.

[2] I. N. Bronstein and K. A. Semendjajew: Taschenbuch der Mathematik. 14. Auflage, Verlag Harry Deutsch, Frankfurt/Main, 1974. (more recent editions available)

[3] Knödel, P.: Rechnerische Ermüdungsnachweise für geschweißte Bauteile. Vortrag in der SLV Mannheim am 26.01.2006. Skript enthalten in den Seminarunterlagen der SLV Mannheim und herunterladbar von www.peterknoedel.de.

[4] U. Kuhlmann: K 4.2 Modellierung und Auslegung von Tragwerken unter ruhender und wechselnder Beanspruchung. Skript Teil IV, Ermüdung und Betriebsfestigkeit, Institut für Konstruktion und Entwurf, Universität Stuttgart, WS 2004 / 2005. (www.uni-stuttgart.de/ke/Lehre/Vorlesungen/K42/Teil4.pdf am 17.01.06)

[5] M.A. Miner: Cumulative damage in fatigue. Journal of Applied Mechanics, Vol. 12 (1945), No. 3, pp 159-164. (cited after [4]).

[6] A. Palmgren: Die Lebensdauer von Kugellagern. Zeitschrift des Vereins Deutscher Ingenieure, Band 58 (1924), Nr. 14, S. 339-341. (cited after [4]).