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Silos with stepped wall thickness on local supports

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Abstract

Some years ago the authors presented an engineering model for the design of cylindrical shells with uniform wall thickness on local supports [4], based on a concept of effective width. In the meantime this model has been improved for the application of silo design.

In this paper numerical studies are presented. The stress field is described which originates from the local support at the bottom of a silo and widens towards the top of the silo. This information helps the engineer to design silos with stepped wall thickness under local loads. Special consideration is given to the hopper junction where additional circumferential stiffness is provided which improves the spreading of the local load.

Keywords: elevated silo, cylindrical shell structure, shell stability, local supports, effective width.

1. Introduction

Although FEA becomes more and more a proven tool for the design of silos it seems that small and medium sized metal silos are still designed ‘by hand’ in most cases. Even if it might not seem reasonable to try to describe a complex situation like the stress field of a silo on local supports by ‘simple’ formulae we see the need for easy tools which help engineers to design silos on local supports.

Some years ago Knoedel and Ummenhofer [4] presented an engineering model for the design of cylindrical shells on local supports. It allows to check the stability by means of the known structural codes by identifying a critical spot above the support and describing the stress concentration at that spot. The stress field above the support is described by means of an effective-width concept where the boundaries of the stress field are given by an ‘angle of load spread’ β. For simplicity uniform wall thickness is assumed and ring stiffeners are considered at the ends of the cylinder only.

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With uniform wall thickness the lowest critical elevation h,cr,el where a buckle can develop is at a height of half of the size of a chess-board half-wave, i.e.

$$h_{,cr,el} = 1{,}7 * \sqrt{(R*T)} \quad (1)$$

above the base of the cylindrical shell – for plastic buckling additional provisions are given. When it comes to real silos with stepped wall thickness it is not sufficient to check the shell at a height of h,cr,el above the support, because every subsequent strake with a smaller wall thickness could become critical, if the concentrated load did not spread wide enough when arriving at this elevation.

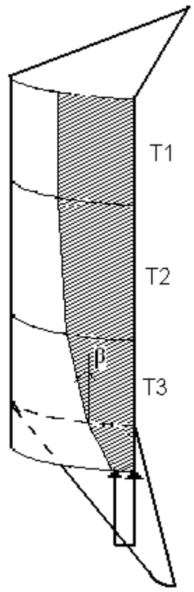

Figure 1: 45°-slice of a silo with stepped wall thickness
and stress field above local support

Therefore we need additional information, such as:

– in which way does stepped wall thickness influence the shape of the stress field;

– in which way does the additional in-plane bending stiffness provided by the joined hopper influence the shape of the stress field;

In the present paper we share our recent results and our experience on that matter.

Other researchers are continuing to investigate this topic on a highly sophisticated numerical level, e.g. Vanlaere et al. [7], Doerich et al. [2], in order to gain a more clear insight in the buckling phenomena, to predict elastic-plastic stability behaviour and to match their experimental results.

2. Method

2.1. Geometry, modelling, loading

The findings presented in this paper are results from linear finite element analyses. For reasons of availability ANSYS 5.3 was used with element SHELL181.

A silo of 4000 mm diameter with 5 mm wall thickness and 10 m length was modelled as basic geometry. The wall thickness of the hopper was chosen to be 5 mm, the thickness of the roof was 3 mm. The meridian of the conical roof was inclined 15° against the horizontal, the meridian of the conical hopper was inclined 30° against the vertical. A base-ring 100x20 was put under the skirt.

Material parameters were taken for steel (Young's modulus $E = 2.1 \cdot 10^5$ N/mm^2) and aluminium alternatively ($E = 0.7 \cdot 10^5$ N/mm^2). Linear elastic material law was assumed without yielding.

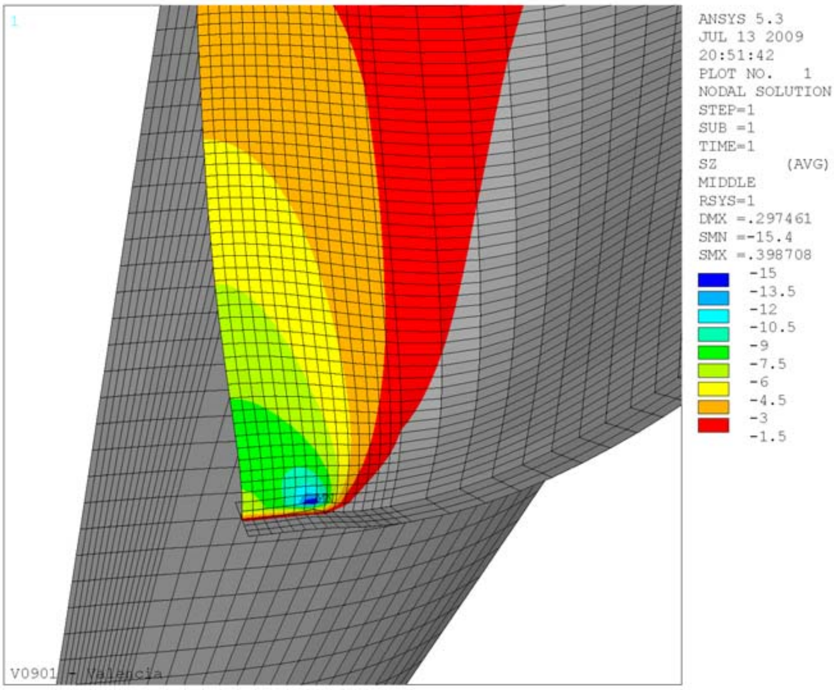

Figure 2: stress concentration factors above support
element size is app. 15/17 mm

The width of the support was chosen to be 250 mm (vertical deformation restraints). Fixing the support radially caused only negligible differences in the results in most cases, producing bigger effective widths. The normalised width of the support is

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$$\mu = n * b_{,support} / circumf = 4 * 250 \text{ mm} / (\pi * 4000 \text{ mm}) = 0.080 \quad (2)$$

From former studies we knew that the effects are very local, so we chose n = 4 supports, used symmetry planes and modelled a 45°-axial-slice of the silo with a circumference of 1571 mm.

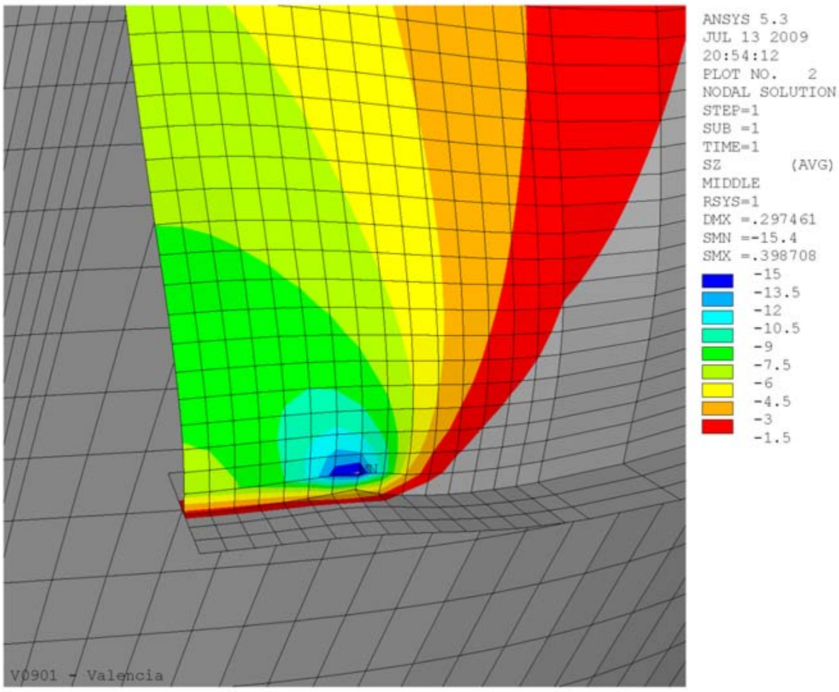

Figure 3: stress concentration factors above support
detail of previous figure

A silo-type surface load was applied to the silo-bin with parameters z0 =7000 mm, maximum horizontal wall pressure phf = 42 kN/m^2, maximum wall friction pwf = 10.5 kN/m^2 at finite depth each, corresponding to a bulk solid with γ = 10.5 kN/m^2, φ = 30°, K = 0.57, μ = 0.25, and following the rules of EC1-4 [1].

2.2. Effective width

In general the concept of 'effective width' is used to transform an arbitrary stress distribution in a structural component into a rectangular block, having the same total load and the same peak stress:

$$b_{,eff}(z) = F_{,total} / sigma_{,peak}(z) \quad (3)$$

with z = height coordinate
of the elevation under consideration.

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The total load was summed up from the restrained nodes of the support. The peak stress was taken from the nodes of the symmetry plane in the middle of the support. This produces erroneous results in the vicinity of the support because immediately above the support the peak stresses are produced by the edges of the support.

In the original study of Ummenhofer [6] the support has been modelled with no deformation constraints in meridional direction but with nodal loads. From a shell stabilty point of view this seemed to be a safe side approach, although rather academic.

The quantity b,eff is assumed to mark the bounds of the stress field. The angle of load spread β is then defined by the inclination of the boundary of the stress field against the vertical. It should be kept in mind however that this concept of effective width assumes stress-free areas outside the stress field which is not realistic but a suitable assumption for 'hand-design'.

3. Global load spread

The shape of the stress field is given in figures 4 and 5. Alternatively to the silo load where the compression in the shell is introduced by frictional loads the silo has been loaded by a line load along the eaves.

It can be seen that the stress field in a silo is much more benign (lines crossing the righthand edge of the diagram) than with an empty axially loaded shell. This is due to the fact that the silo is forced into a radial cos (n*φ)-deformation by the reaction of the n supports; in a filled silo the horizontal loads are straightening this radial deformation, thus helping the meridional peak stresses to spread.

Note in figure 4 that with a modelled circumference of 1571 mm of the 45°-segment the effective width should not exceed this value. For a shell with silo loads the total load decreases from bottom to top. So a more rigorous definition of effective width would not use the total support reaction, but the load for that elevation, for which the effective width is sought, having

$$b,eff(z) \;=\; F(z) \,/\, sigma,peak(z) \qquad (4)$$

However we did not use this more accurate procedure, assuming that the designer wants to take the force which he determined as support load (including vertical components due to wind, snow, out-of-plumbness, seismic action, and so forth) and see whether he can pass all checks with this force.

Downsizing of the wall thickness at higher strakes does not seem to affect the angle of load spread β. According to Knoedel/Ummenhofer [4] the angle should decrease for increasing R/T-ratios, having a break at each step of a wall thickness.

The bounds of the stress field depend on the diameter of the silo rather than on the silo's height. In figure 4 the lines for the 10-m-silo and the 20-m-silo fall close together.

Note in figure 5 that (from bottom to top) the stress field's width seems to decrease to a minimum of app. 200 mm and then increases again. This effect indicates that close to the support the peak stresses do not lie on the central meridian of the support. If the top part of

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the line would be prolonged slightly curved downward, it would match the x-axis at about 125 mm which would be reasonable.

The angle of load spread for the empty shell can be described as (see figure 4)

$$\beta_{,global,empty,z\leq R} = \text{arc tan}[(600-125)/2000] = 13.4° \quad , \tag{5}$$

which holds for $z \leq R$. Beyond that the angle halves to

$$\beta_{,global,empty,z>R} = \text{arc tan}[(1800-600)/(13000-2000)] = 6.2° \tag{6}$$

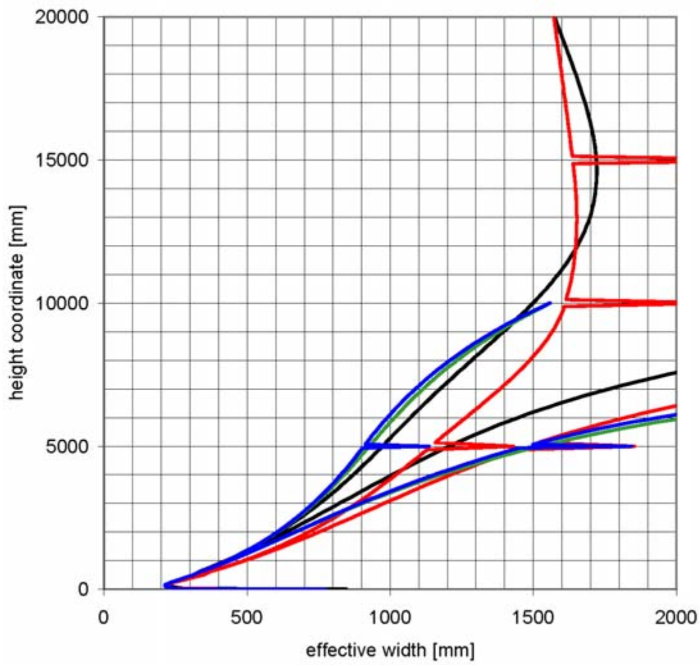

Figure 4: Effective width along the silo's height
black, green: 5 mm continuous wall thickness
red, blue: 10-8-6-4 mm stepped wall thickness

The angle of load spread for the filled shell can be described as (data for the effective width taken from the 20-m-silo)

$$\beta_{,global,filled} = \text{arc tan}[(1555-125)/6375] = 12.6° \quad , \tag{7}$$

which holds until the stress field reaches the axis of symmetry.

Mark that the prediction for $R/T = 400$ and $\mu = 0{,}080$ read from figure 3 in Knoedel / Ummenhofer [4] is $\beta_{,local}$ = app. 14°.

4. Hopper junction

It is well known that the ability of a cylindrical shell to spread local meridional loads is governed by it's circumferential bending stiffness (see e.g. Greiner's formulae on semi-membrane theory [3]). The hopper junction provides a considerable amount of circumferential bending stiffness by the effective width of the hopper itself as well as parts of the bin and the skirt and possibly an additional ring girder at the transition junction. Knoedel and Ummenhofer assumed earlier that between stiffening rings an angle of load spread of β = 30° could develop but there is no scientific background for this assumption up to date.

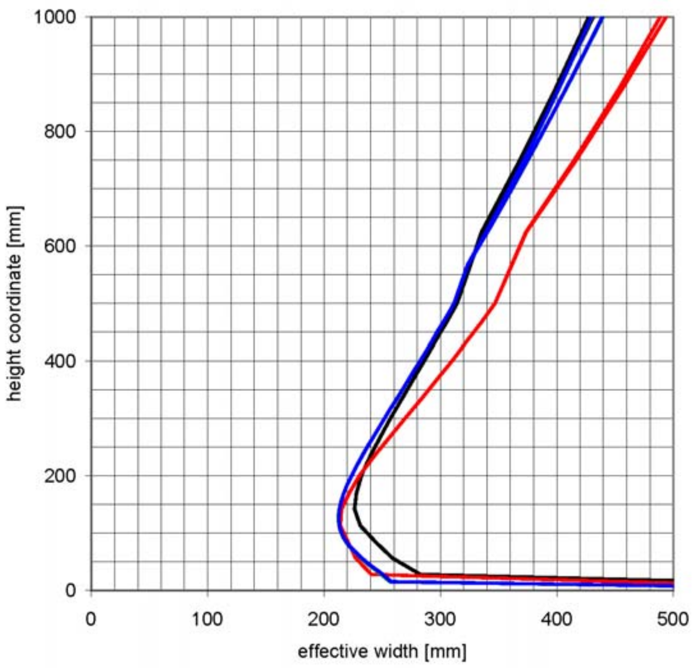

Figure 5: Effective width along the silo's height
Detail above support (green line is hidden behind black and blue line)

Many designers allow for an angle of β = 45° – without scientific background as well – and there is only a little number of silo failures which can be contributed to this assumption.

The influence of the hopper junction was studied at an empty cylinder with silo loads and eaves loads alternatively, the support nodes were not fixed radially. The definition of the angle β corresponds to eqs 5 et seq., describing the inclination of the bound of the stress field against the vertical, beginning at the (right) edge of the support. The angle β is evaluated for z = h,cr,el = 170 mm as given in eq 1, to provide some information on possible local failure according to Knoedel and Ummenhofer [4]. A second evaluation is done for z = z,junction, to provide information on the width of the stress field above the hopper junction. For the evaluation the effective width one element below is used because at z,junction itself there is an indent in the course of the effective widths due to averaging bin and hopper elements.

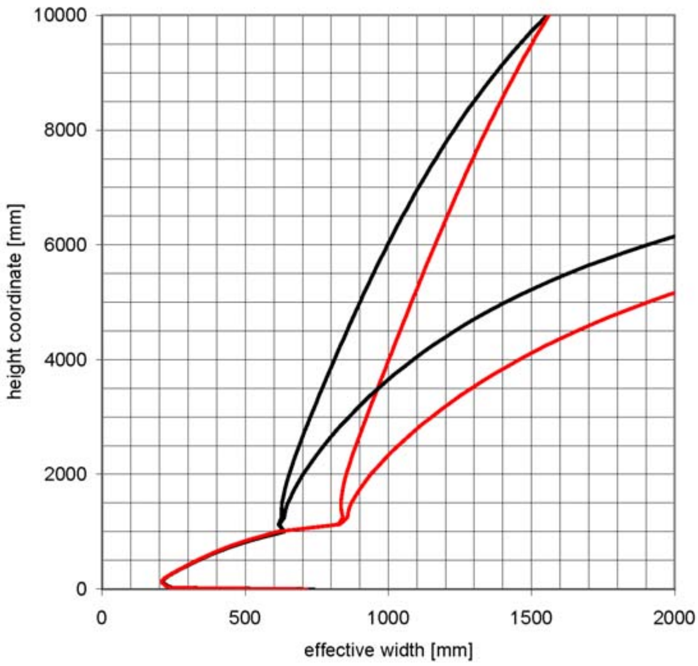

Figure 6: Effective width along the silo's height, hopper attached at z = 1000 mm
black: hopper only; red: ring stiffener with infinite stiffness

It can be taken from figures 6 and 7 that the additional stiffness of the hopper increases the angle of load spread to

$$\beta\text{,global,hopper} = \text{arc tan}[(625–125)/1000] = 26.6° \qquad (8)$$

within the skirt. If the transition provides enough circumferential stiffness the load spread within the skirt remains the same, but above the transition the effective width jumps to 800 mm, giving an effective angle of load spread of

$$\beta\text{,global,stiff} = \text{arc tan}[(800–125)/1000] = 34.0° \qquad (9)$$

for the design of the strakes above the transition.

For the stress field above the transition it may be assumed for the empty shell that the stress field spreads linear to the width of the symmetry-slice when arriving the eaves In the above example this would incorporate an error of 10–15% to . For the filled shell it may be assumed that the stress field spreads within a height of one diameter which corresponds roughly to an angle of

$$\beta\text{,global,above\_hopper} = \text{arc tan}[(1600–600)/(5500–1000)] = 12.5° \qquad (10)$$

which happens to be close to the above findings for filled shells.

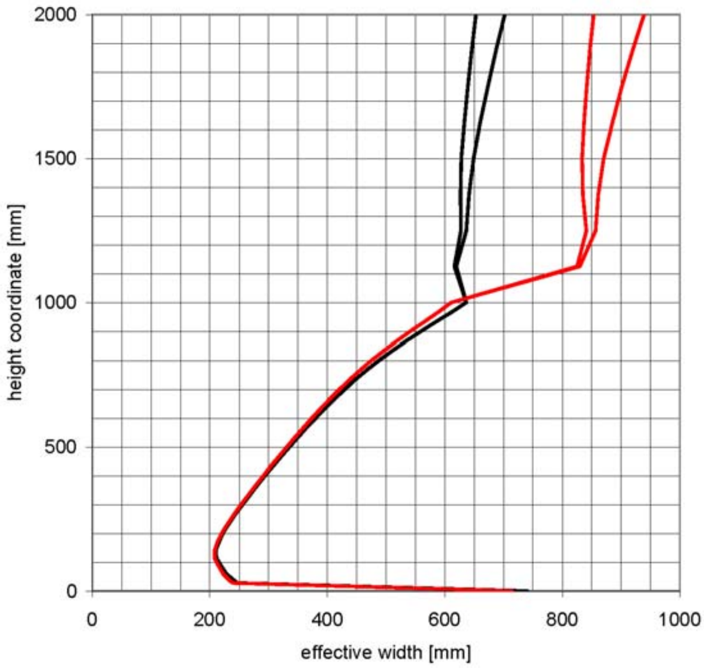

Figure 7: cut out from figure 6
element size below transition app. 30 mm, above transition app. 125 mm

Figure 8 shows the effect of different skirt length. The local situation, i.e. the effective width at h,cr,el = 170 mm is only very little affected by the length of the skirt and by the type of support (radially restrained or not). The effective width at the transition depends strongly on the type of support. An optimistic guess for a skirt length of 1000 mm is

$$\beta\text{,global,skirt,opt} = \text{arc tan}[(1200–125)/1000] = 47.1° \quad , \qquad (11)$$

a pessimistic guess would be

$$\beta\text{,global,skirt,pess} = \text{arc tan}[(625–125)/1000] = 26.6° \quad , \qquad (12)$$

a value which was given in eq. 8 already.

With real silo structures on local supports two different types of supports are imaginable:

a) If the silo is elevated on columns these would not provide any radial stiffness, so a 'radially free' support would be the proper assumption.

b) If the silo has supports above load cells or brackets which are connected to a steel substructure, the clearance and the horizontal flexibility of the substructure should be taken into account. In our calculations the displacement of the 'free' supports was in the order of 6 mm. This corresponds more to a flexible steel substructure than to a radially stiff support. Therefor we recommend to assume 'free' in both cases.

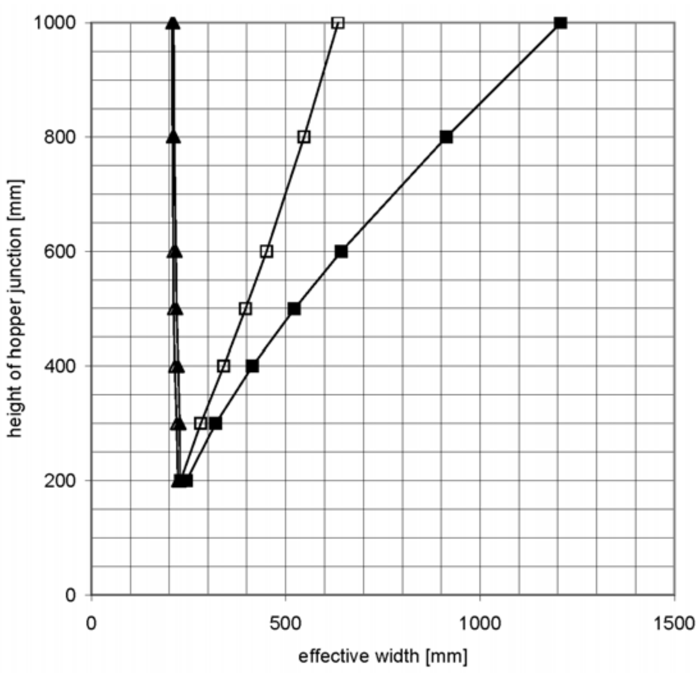

Figure 8: Effect of skirt length
square: b,eff at transition; triangle: b,eff at h,cr,el
filled: support radially fixed; hollow: support radially free

5. Summary

– Designing silos on local supports is a difficult task. Typically the load is introduced into the structure at the supports at only 8 % of the circumference. For an example of a silo with R/T = 400 some features have been described, as how the shell spreads the concentrated load around the circumference.

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– Circumferential stiffness is the most effective feature which the shell should have. Therefore a base ring with sufficient radial stiffness is important.

– The load spreading behaviour of a filled shell is much more benign than the predicitons of Knoedel and Ummenhofer [4] for empty shells.

– With a ring of sufficient stiffness at the hopper junction an effective angle of load spread of 45° can be reached in the skirt.

– The above results are identical for both aluminium and steel.

6. Open questions and future work

– In the present study the base ring was kept free to rotate about the circumference, only the nodes under the skirt were supported. For practical applications you should expect the ring to remain horizontal at the support – except for having a load cell with a literal point load.

– In the present study the peak stress for the evaluation of the effective width was taken from the centerline of the support. As stated above the maximum stress could be in another spot.

– In the present study the hopper was only treated with regard to its stiffness. The hopper remained unloaded and thus exerted no stress resultants into the skirt.

– There is buckling failure typically in the upper part of the silo adjacent to a step in wall thickness which might be connected to local loads with insufficient load spread (Knoedel [5]). In the present study we found situations where there was a little stress peak in some distance of the meridian above the support. However this peak exceeded the surroundings only by 1-2 N/mm^2 which is no strong indicator to trigger buckling.

– The study should be extended to annular rings of finite stiffness and practical size.

– A parametric study could be done to investigate other R/T-ratios and different widths of support.

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References

[1] EN 1991 (EC1): Eurocode 1: Actions on structures. Part 4:2006: Silos and tanks.

[2] Doerich, C., Vanlaere, W., Lagae, G., Rotter, J.M., Stability of column-supported steel cylinders with engaged columns, in *Proceedings of the IASS Symposium 2009*, Domingo, A. and Lazaro, C. (eds.), 28.09.-02.10.2009, Universidad Politecnica de Valencia, Spain.

[3] Greiner, R., Zur Längskrafteinleitung in stehende zylindrische Behälter aus Stahl. *Stahlbau*, 1984; **53**; 210-215.

[4] Knödel, P. and Ummenhofer, T., Ein einfaches Modell zum Stabilitätsnachweis zylindrischer Schalentragwerke auf Einzelstützen. *Stahlbau,* 1998; **67**; 425-429.

[5] Knoedel, P.: Recent Silo Codes – and still Structural Failure? pp 113-122 in *Structures and Granular Solids – From Scientific Principles to Engineering Applications. An international conference in celebration of the 60th birthday of Prof. J. Michael Rotter*, Chen, J.F., Ooi, J.Y., Teng, J.G. (eds), The Royal Society of Edinburgh, Scotland, UK, 1-2 July 2008. Taylor & Francis Group, London 2008.

[6] Ummenhofer, T., Numerische Untersuchungen zur Lasteinleitung in Zylinderschalen. Diplomarbeit bei Prof. Mang, Versuchsanstalt für Stahl, Holz und Steine, Universität Karlsruhe 1990. (Numerical investigations on local loads in cylindrical shells. Diploma thesis, in German)

[7] Vanlaere, W., Sonck, D., Callewaert, D., van Impe, R., Lagae, G., The buckling behaviour of steel cylinders with engaged columns, in *8th National Congress of Theoretical and Applied Mechanics*, Brussels, 28.-29.05.2009.

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