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IMPERFECTIONS IN METAL SILOS

MEASUREMENT, CHARACTERISATION

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STRENGTH ANALYSIS

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Substitute Imperfections for the Prediction of Buckling Loads in Shell Design

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Abstract

For design purposes many attempts have been made to allow for characteristic imperfections in shell stability analysis. Due to the different tools available, i.e. classical analysis, numerically solved equation systems, FEM, imperfection patterns of different complexity have been incorporated in the problem description. After presenting some widely used characteristic patterns a two level approach is proposed for design purposes.

1 Introduction

Imperfections are known to be the reason for the difference of the performance of real thin walled shells under compressive loading compared to the critical load of an ideal shell.

Nowadays, powerful numerical tools are available which allow for arbitrary shell shapes, boundary conditions and loading as well as geometrical and material nonlinearities. Simple rational substitute imperfections to be introduced into shell analysis seem to be the missing link towards a realistic prediction of shell performance.

This paper outlines the state of the art in shell imperfections and proposes some design rules.

2 Substitute Imperfection Patterns

2.1 History

According to *Krysik*'s survey of the literature (1994 [28]) *Mallock* 1908 [32] and *Lilly* 1908 [29] were the first to 'discover' (local) buckling phenomena in experiments with cylindrical shells. At the same time *Lorenz* 1908 [30] did the first theoretical investigations in this topic. In 1932 however *Fluegge* [13] seemed to be the first to realize the remarkable difference between the 'classical solution' for the critical stress (eq. 1) of a cylindrical shell under axial compression and experimental results.

$$\sigma_{\text{cl}} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{T}{R} \approx 0.605\ E\ \frac{T}{R} \tag{1}$$

From that time on a long list of researchers tried to incorporate reasonable geometrical imperfections into their analyses and still the buckling loads obtained in shell tests were lower.

$$\sigma_{\text{cr,imperfect}}^{\text{theoretical}} > \sigma_{\text{u}}^{\text{experimental}} \tag{2}$$

This fact motivated the 'quest of worst case imperfections' – the aim was to *knock down* the findings of the classical buckling analysis. The imperfection patterns used were strongly dependent on the tools available and on the type of analysis which could be handled as easy as possible. Harmonic sine/cosine patterns were used as substitute imperfections in analyses 'by hand' and eigenmodes were used in numerical analyses.

On the other hand it was understood, that real imperfections are rather non-harmonic, non-eigenmode patterns with random features. It was suspected, that the weld shrinkage at circumferential welds plays a key role in the performance of shells in civil engineering.

Some widely used models are presented in the following. For simplicity empty cylinders under pure axial compression are considered only.

2.2 Model I: Regular Harmonic Pattern

The first imperfection pattern to be used were harmonic waves in circumferential and longitudinal direction of the shell (e.g. *Donnell* 1934 [6], *v.Karman/Tsien* 1941 [19], *Donnel/Wan* 1950 [7]). *Thielemann/Dreyer* 1956 [37] used a modified formulation for the buckling mode, which was used as shape funktion for the initial imperfections as well:

$$w = a_0 \left[\cos\frac{\pi x}{l_x} \cos\frac{\pi y}{l_y} + b \cdot \cos\frac{2\pi x}{l_x} + c \cdot \cos\frac{2\pi x}{l_y} + d \right] \tag{3}$$

Based on results of *Donnell* some of the quantities were fixed to the following values: $l_y/l_x = 0.728$ (ratio of circumferential to longitudinal half wave length); $b = 0.18$; $c = 0.03$; $d = 0$.

Remark: It is known that the 'diamond buckles' are wider in circumferential direction. So it should read $l_x/l_y = 0.728$ instead.

This pattern was supposed to represent the critical eigenmode. It shows the remarkable decrease of the ultimate load of the shell even for very small imperfections (see fig. 1). For imperfection amplitudes of $w_0/T \geq 0.4$ the structure shows no critical or

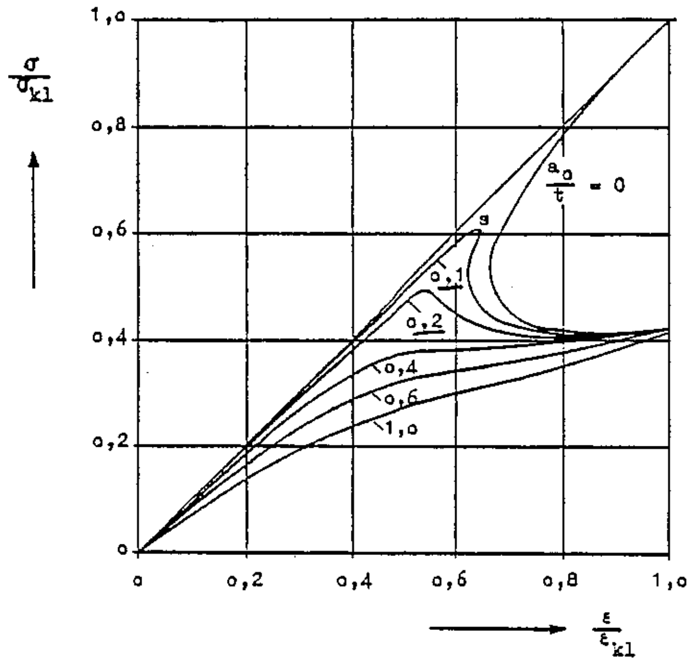

Figure 1: Stress–Strain Diagrams of Elastic Cylinders under Axial Compression (from *Thielemann/Dreyer* 1956 [37])

ultimate load. The axial stiffness is so much reduced, that the load–deflection–path goes continuously into the 'postbuckling' state. From the experience with imperfections of real shells and buckling tests of model shells it was clearly felt however, that these results were much too pessimistic to be used for design purposes.

2.3 Model II: Meridional Multiwave – Axisymmetric Imperfections

A recent literature survey on theoretical work on axisymmetric imperfections was done by *Werner* 1993 [40]. He compared most of the known studies and tried to find unique information on the imperfection sensitivity of a shell with $R/T = 1000$. A sample of the work is presented in this section (axisymmetric. along the meridian multiple-wave imperfections) and in section 2.4 (single axisymmetric imperfections).

Axisymmetric bulges which are regular sinusoidal in meridional direction were investigated by *Koiter* 1963 ([26], [27]), *Hutchinson* 1965 [16] and *Teng/Rotter* 1992 [36].

The meridional shape is chosen to be (notation of *Hutchinson*)

$$\begin{aligned} W_0/T &= -\bar{\xi}_1 \cos \frac{q_0 x}{R} \\ \text{where } \bar{\xi}_1 &= w_1/T \end{aligned} \tag{4}$$

and the critical number of full waves (*Koiter*) along the meridian is given by

$$q_0^2 = R/T \cdot \sqrt{12(1-\nu^2)} \tag{5}$$

In that way the shape of the imperfection corresponds to the critical wavelength

$$\begin{aligned} \lambda^2_{\mathrm{cr,N=0}} &= \frac{4\pi^2 \cdot R \cdot T}{\sqrt{12(1-\nu^2)}} \\ \lambda_{\mathrm{cr,N=0}} &\approx 3.456 \cdot \sqrt{R \cdot T} \qquad (\text{for } \nu = 0.3) \end{aligned} \tag{6}$$

of the axisymmetric eigenmode of the perfect shell and is by a factor of $\sqrt{2}$ smaller than the bending full wave

$$\begin{aligned} \lambda^2_{\mathrm{bend}} &= \frac{4\pi^2 \cdot R \cdot T}{\sqrt{3(1-\nu^2)}} \\ \lambda_{\mathrm{bend}} &\approx 2 \cdot 2.444 \cdot \sqrt{R \cdot T} \qquad (\text{for } \nu = 0.3) \end{aligned} \tag{7}$$

Teng/Rotter simulated the analytical solutions of *Koiter* and *Hutchinson* by FEM. They found that a multi wave pattern with a minimum of 9 consecutive meridional half waves gives the same results like a shell of infinite length with an infinite number of axisymmetric imperfections. The 'knock down factors' to be expected with this type of imperfection is shown in fig. 2. Mark that δ_0 denotes the double amplitude of the imperfection used in the FEM.

Axisymmetric bulges which are irregular in meridional direction were investigated by *Knoedel* 1995 [25]. He used the meridional shape which corresponds to the lowest critical eigenvalue coupled to an axisymmetric eigenmode (and accounts for the boundary conditions), and obtained a buckling coefficient $\alpha = \sigma_{\mathrm{cr}}^{\mathrm{imp}}/\sigma_{\mathrm{cr}}^{\mathrm{perf}} = 0.27$ ($R/T = 1000$; $w_0/T = 1$). Since the imperfection pattern used cannot be described *a priori*, but is a result of the nonlinear prebuckling calculation (allowing for boundaries, material and geometrical nonlinearities) the pattern was denoted as a *procedural imperfection*.

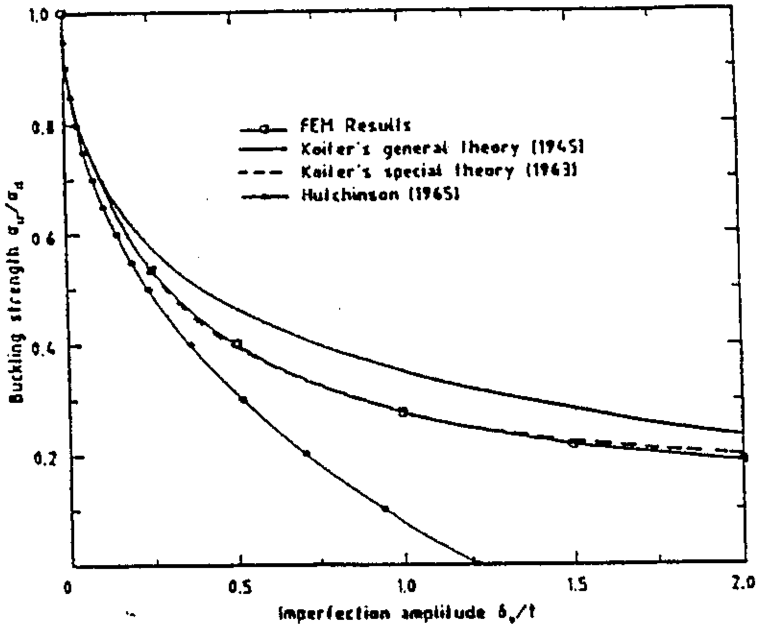

Figure 2: Results from *Teng/Rotter* and other authors (from [36])

2.4 Model III: Single Axisymmetric Bulge

In 1963 *Fischer* [12] published results for a single axisymmetric imperfection which has a normalised depth of α and which has a dimensionless length of 2ε along the meridian. The shape of the imperfection is given by

$$\begin{aligned} \omega_o &= \alpha cos^3 \frac{\pi}{2}\frac{\xi}{\varepsilon} \\ \text{where } \xi &= \frac{x}{2\pi R} \\ \text{and } w &= \omega\sqrt{\frac{D}{E \cdot T}} \\ \text{and } D &= \frac{E \cdot T^3}{12(1-\nu^2)} \end{aligned} \qquad (8)$$

Typical results for $R/T = 800$ and $L/R = 0.865$ are shown in fig. 3. The upper curve ($\varepsilon = 0.0688$) is obtained with a single half wave along the meridian. The lower curve ($\varepsilon = 0.02$) is obtained with an imperfection size which corresponds to the bending double half-wave of the shell.

Babcock/Sechler 1963 [3] used an axisymmetric imperfection, which has one sine half wave along the meridian. Since this does not seem to have relevance for civil engineering structures, it will not be discussed in this place.

Juercke/Kraetzig/Wittek 1983 [17] investigated an elastic shell ($R/T = 1428$; $L/R = 3$) with a toroidal imperfection. The supported edge was pinned, the loaded edge was free. The meridional shape of the axisymmetrical bulge consisted of four equal

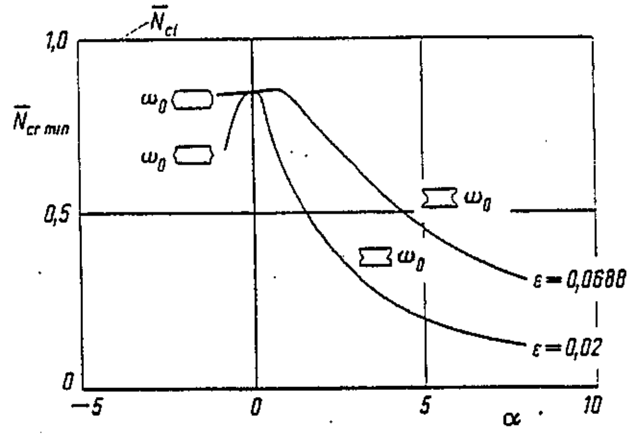

Figure 3: Results of *Fischer* [12], $w_0/T \approx 0.303\alpha$

sized circular sections, the full wave length was chosen to be

$$l_m = L/10 \approx 11.3\sqrt{R \cdot T} \tag{9}$$

which is roughly 2.3 times greater than the bending wave length. For an imperfection depth of $w_0/T = 4.3$ the buckling coefficient was found to be

$$\alpha = P_{w_0/T=4.3}/P_{w_0/T=0} = 0.330 \tag{10}$$

Teng/Rotter [36] studied the effect of different single axisymmetric imperfections (compare section 2.3). They found that single imperfections give some 25% higher buckling loads for $w_0/T = 1$ than the above multi wave pattern (see fig. 4).

2.5 Model IV: Eigenmode

Imperfections in the shape of the eigenmode of the perfect structure have been used by many authors. In a first step the (critical) eigenmode of the perfect struture is determined by an eigenvalue analysis. In a second step the normalised deflections are scaled and superimposed onto the original perfect structure. The axisymmetric 'procedural imperfection' used by *Knoedel* [25] (non-critical mode, see section 2.3) is a special case of this imperfection model.

Eckstein 1983 investigated a cylindrical panel (using symmetry conditions) of approx. 7° angle of aperture which is about 1/50 of the circumference. With an inward orientated imperfection in the shape of the first eigenmode the ultimate load of the shell drops to 0.776 of the perfect shell, when the imperfection depth is set to only $w_0/T = 0.05$.

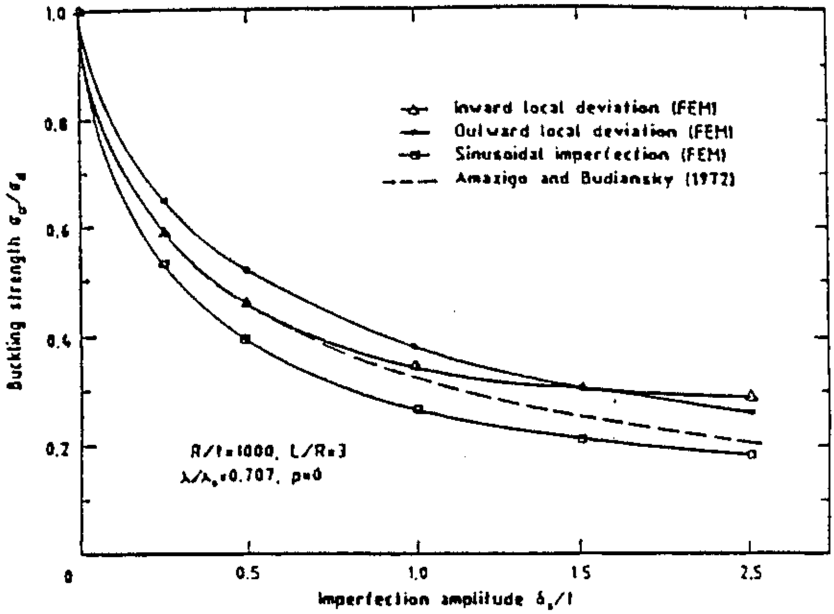

Figure 4: The effect of different axisymmetric imperfections (from *Teng/Rotter* [36])

Wunderlich and co-workers published many studies on imperfect shells, mainly with dished ends under internal pressure ([42], [43]). These are cited here for reasons of completeness and are not discussed in detail.

2.6 Model V: Local Buckle

There are only few studies where a single local imperfection is used. *Babcock* [2] presented 1968 results of investigations. A further study was recently published by *Knebel/Schweizerhof* [20] where a local inward deviation in a sinusoidal shape is generated in a 36° section of a cylindrical shell. Figure 5 shows the results of the finite element calculations.

There seems to be a relation between cutouts in shells and local imperfections. The modelling of single local imperfections needs a lot of computational time because the whole shell has to be generated. This seems to be the reason for the few investigations which were undertaken.

Schulz postulated the equivalence of a cutout in the shell wall and a local imperfection of infinite depth [35]. Although this does not hold with respect to the stress-strain state (there are no section forces perpendicular to the edges of the cutout whereas a deep buckle induces deviating forces into the shell) it can be used to discuss lower bounds of the buckling strength of a shell. With experiments on cylinders

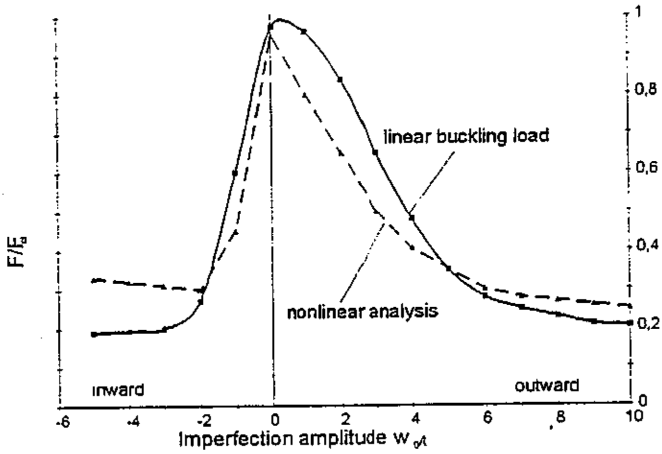

Figure 5: Influence of imperfection depth (*Knebel/Schweizerhof* 1995 [20])

with quadratic cutouts under pure bending *Knoedel/Schulz* 1988 [21] found a lower bound of the buckling stress of the meridional free edge of the cutout. They assumed that there is a limit cutout size, beyond which no further reduction of the buckling stress will appear.

With this idea transferred to imperfections one can conclude from *Knoedel*'s experiments 'series A' [25]: Even with big radial imperfections (although within the ECCS tolerances) and very poorly manufactured rims (gaps of as much as 3 mm between adjacent parts of the shell are reported) the buckling stress does not fall below the DIN 18800 curve (close to the ECCS regulations).

2.7 Model VI: Irregular Pattern

Arbocz/Babcock [1] determined some lower harmonics in a two dimensional *Fourier*-decomposition of the imperfections of the shell. By application of the 'Multimode Analysis' different combinations of harmonic imperfection patterns associated with characteristic amplitudes were introduced. The numerical and experimental results were compared and the combination which results in buckling loads closest to the exprimental results was determined.

Real irregular imperfection patterns were investigated by *Ummenhofer* [39] who generated the shape for the finite element model directly by use of the measured grid of surveyed specimens (compare [38]). The influence of the maximum imperfection depth on the buckling coefficient is shown in fig. 2.7 where max δ_0 is the maximum deviation against the template or rod of ECCS R4.6 fig. 2 [8].

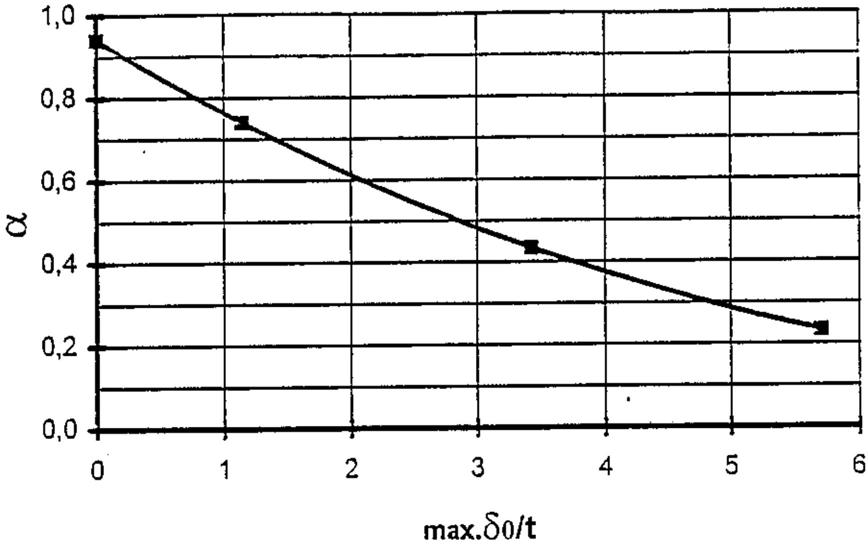

Figure 6: Influence of the imperfection depth on the buckling coefficient of a shell with random imperfections and clamped edges (from *Ummenhofer* 1996 [39])

3 Substitute Imperfections for Engineering Purposes

3.1 Procedure Required

The numerical procedure required for the design of thin walled shells is as follows:

- generate the shell structure
- impose the below imperfections on the level required/desired
- impose design loads, allow for loading imperfections if required/desired
- calculate the load–deflection path, allow for geometrical nonlinearity
- allow for material nonlinearity if this is required by the code used (depending on geometrical and material parameters)
- search for bifurcation or ultimate loads whichever are lowest

The amplitude of the substitute imperfection may be chosen such that besides the real geometrical imperfections the effects of all other imperfections are covered, such as material inhomogenities, residual stresses, loading imperfections (this is discussed in further detail below).

3.2 Level I: Single Axisymmetric Depression

Theoretical and numerical studies show, that axisymmetric imperfections are most detrimental to the structural behaviour of the shell. Therefore it seems sufficient. to use a single axisymmetric imperfection in design. Based on the results of *Teng/Rotter* the half wave length should be closest to

$$\lambda_{\mathrm{subst},N=0} = 2.44\sqrt{R \cdot T} \tag{11}$$

In choosing the imperfection amplitude two aspects have to be observed:

- A *manufacturing limit imperfection depth* has to be chosen according to the process of manufacturing and the abilities of the manufacturer. According to a proposal of *Rotter* [33] which is based on the current ECCS regulations

  $$(w_0/T)_{lim} = \frac{1}{25}\sqrt{R/T} \tag{12}$$

  may be used for 'quality construction', 1/16.5 would be 'poor', 1/40 would be 'excellent'. The chosen manufacturing limit imperfection depth has to be proved on the completed shell by means of the ECCS template.

- The *design imperfection depth* which is needed to calibrate the imperfection pattern used in the calculations has to be chosen greater than the manufacturing limit imperfection depth, if it shall cover the effects of residual stresses, loading imperfections and so on. According to *Esslinger* [11] the effect of the radial imperfections along with residual stresses account for 60% of the reduction of the buckling load. 40% account for uneveness of the supported edges and loading imperfections. Further discussion can be found in *Knoedel/Ummenhofer/Brenner* 1994 [24]. We assume that the total effect of imperfection is composed as follows: 50% radial imperfections: 10% residual stresses; 40% support and loading imperfections and other effects. Assuming that the imperfection depth is proportional to the reduction of the buckling load the design imperfection may be chosen as

  $$(w_0/T)_d = \frac{100\%}{50\%}(w_0/T)_{lim} = 2 \cdot (w_0/T)_{lim} \tag{13}$$

This imperfection model covers silo shells and other civil engineering structures, where strakes are joined by circumferential welds. In this case, the radial random imperfections of the strake outside the region of the weld depression have much less effect, so they do not need to be taken into consideration.

3.3 Level II: Random Imperfections

If a shell structure is stiffened by ring stiffeners, the detrimental effect of the circumferential weld is counteracted by the ring stiffener. In this case it is sufficient to impose random imperfections, which reduce the buckling strength much less than the above axisymmetric imperfections. This holds for shell structures as well, which do consist of a single strake only.

For numerical calculations one has to generate a random imperfection pattern which exhibits features as described by *Ummenhofer/Knoedel* ([38] section 3.3 fig. 10). For specific manufacturing techniques the *Fourier*-harmonics of the coefficients A, B, C, D might be approximated by simple relations.

The maximum normalised deviation should be chosen to be equal to the design imperfection depth above.

3.4 Loading Imperfections

If loading imperfections such as uneven edge loads or unintended edge moments can be described, they do not have to be accounted for separately by increasing the design imperfection depth as shown above.

This holds for edge moments which are induced by misalignment of adjacent shell strakes (compare *Knoedel/Maierhoefer* 1989 [23]). Another typical example would be an elevated silo, where the load concentrations due to the single columns are modelled realistically in the numerical model (compare *Knoedel/Ummenhofer/Brenner* 1994 [24]).

3.5 Material Imperfections

Material imperfections such as an orthotropic distribution of *Young*'s modulus or inhomogenities in the wall thickness or yield strength with steel shells have comparatively little effect, so they can be neglected in design.

4 Conclusions

Different models of substitute imperfections have been discussed and compared. Main difference is the effort, which has to be undergone when applying the different models. Therefore a two-level approach has been proposed for the design of silo shells.

On Level I a simple axisymmetric imperfection is used, which can be introduced into the FE model with little effort. With this imperfection buckling loads are obtained, which match about the lower bound of the known experiments.

On Level II a random imperfection is used, the shape of which corresponds to individual techniques of manufacturing single strakes. This imperfection may be used if there is no circumferential weld or the negative effects of depressions due to weld shrinkage are counteracted by ring stiffeners. The buckling loads obtained may be clearly higher than the lower bound of the experimental evidence.

The imperfections on both levels may be calibrated to the quality of the workmanship of the producer. Thus, it would be possible to design for less imperfections and subsequently higher buckling loads, if shallower imperfections can be controlled by a more sophisticated manufacturing technique.

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