

Knoedel/Ummenhofer at Eurosteel Copenhagen

Mass Variation with dissipative Steel Structures under Seismic Loads

Background Document on FE Study

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1 Introduction

2 Terms

DAF	dynamic amplification factor: (max response) / (driving amplitude)
DBF	design by formula, i.e. hand calculation
dot	used as thousands separator
comma	used as decimal point
NINC	number of load steps within one period; 200 is used in this study
	NSUB,100,10000,10 was used to
	start with 100 subdivisions of each load step (= 0,08 ms with a 1,6-s-period)
	have a smallest time step of 1 period over 200*10.000 (= 0,0008 ms with a 1,6-s-period)
	accelerate to a time step of not larger than 1 period over 200*10 (= 0,8 ms with a 1,6-s-period)
	Remark 1: This seems very careful, but helps to prevent the system from diverging after some while
	Remark 2: due to the acceleration the system starts bisecting every time a new peak is reached. Therefore a further acceleration does not seem to be effective.

3 Model

3.1 FE Package

ANSYS R16.0 Academic – APDL, implicit

3.2 Geometry

Frame $w/h = 6 \text{ m} / 4 \text{ m}$

see previous studies, e.g. Tampere 2015 and Napoli 2014

For the single storey frame we used columns 240x12 and a waler 1000x12, the cross sectional properties are given below.

3.3 Masses

36.000 kgs in each corner of the frame, giving a total storey mass of 72.000 kgs.

3.4 Elements

We used a 2-D ANSYS model with plastic BEAM23. For simplification all sections were chosen to be tubular so we can follow the structural needs without being restricted by the stepped dimensions of a profile table.

Shortcoming of this element is, that there are only 5 layers along the height of the element.

A length of 0,20 m i.e. 20 elements for the columns and 30 elements for the waler proved to be sufficient for the present analysis (Knoedel/Ummenhofer 2015). In the present study, an element length of 0,10 m was used, so that we have 40 elements in the column and 60 elements in the waler.

3.5 Material

We used steel with a simplified bi-linear constitutive law with isotropic hardening.

Young's modulus with 2,1 GPa was used up to a yield limit of 235 MPa. Above that we used a straight line to the ultimate tensile stress 360 MPa at assumed 20 % strain. Thus we receive an 'engineering plastic modulus' with a slope of $(360 \text{ MPa} - 235 \text{ MPa}) / 0,20 = 625 \text{ MPa}$

This approach has been used before, see eq. 15 in Knoedel/Hrabowski (2012).

3.6 Damping

A parameter DAMP = 0,00235 was used to get

see Run_7: $\delta = 0,0292 / 0,0290$

see Run_13: $\delta = 0,0413 / 0,0415$

see Run_18: $\delta = 0,0208 / 0,0210$

The damping ratio (Lehrsches Dämpfungsmaß D) amounts to

$$\zeta = \delta / 2\pi$$

see Run_7: $\zeta = 0,004647 / 0,004615$

see Run_13: $\zeta = 0,006573 / 0,006605$

see Run_18: $\zeta = 0,003310 / 0,003342$

3.7 Verification

3.7.1 DBF

Elastic limit moment of column

$$M_{\text{lim,el}} = 115,2 \text{ kNm}$$

Horizontal elastic limit force (2 columns – 1 storey)

$$H_{\text{lim,el,storey}} = 2 * 115,2 \text{ kNm} / 4 \text{ m} = 57,6 \text{ kN}$$

Horizontal plastic limit force ($\alpha_{\text{plastic}} = 1,27$ for thin-walled tubulars)

$$H_{\text{lim,pl}} = 57,6 \text{ kN} * 1,27 = 73,1 \text{ kN}$$

Horizontal storey-stiffness with infinitely stiff waler

$$c = 2 * 3 * E * I / L^3$$

$$c = 2 * 3 * 2,1 * 10^5 \text{ MPa} * 5585 \text{ cm}^4 / (4 \text{ m})^3 = 1100 \text{ N/mm} = 1,10 * 10^6 \text{ N/m}$$

Storey drift

$$u_{\text{lim,el}} = H_{\text{lim,el}} / c$$

$$u_{\text{lim,el}} = 57,6 \text{ kN} / 1100 \text{ N/mm} = 52,4 \text{ mm}$$

Eigenperiod

$$T = 2\pi * \sqrt{(m/c)}$$

$$T = 2\pi * \sqrt{(72.000 \text{ kgs} / 1,10 * 10^6 \text{ N/m})} = 1,6075 \text{ s}$$

Eigenperiod with model stiffness

$$T = 2\pi * \sqrt{(72.000 \text{ kgs} / 1,0868 * 10^6 \text{ N/m})} = 1,6172 \text{ s}$$

3.7.2 Model

see Run_8

Horizontal stiffness

$$c = 57,6 \text{ kN} / 53,0 \text{ mm} = 1,0868 \text{ kN/mm}$$

Elastic limit horizontal load should be corrected to

$$57,6 \text{ kN} * 235 \text{ MPa} / 240 \text{ MPa} = 56,4 \text{ kN}$$

$$\dots \text{ and a displacement of } 53,0 \text{ mm} * 235 \text{ MPa} / 240 \text{ MPa} = 51,9 \text{ mm}$$

Eigenperiod see Run_7

$$T = 1,6170 \text{ s} \dots \text{ fits wonderful}$$

4 Properties

4.1 CHS 240x12

$$R = (24 \text{ cm} - 1,2 \text{ cm}) / 2 = 11,4 \text{ cm}$$

$$I = \pi * (11,4 \text{ cm})^3 * 1,2 \text{ cm} = 5585 \text{ cm}^4$$

$$W = 5585 \text{ cm}^4 / 11,4 \text{ cm} = 490 \text{ cm}^3$$

$$A = 2\pi * 11,4 \text{ cm} * 1,2 \text{ cm} = 85,6 \text{ cm}^2$$

For information:

HEA 240:

$$I = 7760 \text{ cm}^4$$

$$W = 645 \text{ cm}^3$$

$$N_{R,k} = 85,6 \text{ cm}^2 * 235 \text{ N/mm}^2 = 2010 \text{ kN}$$

$$M_{lim,el} = 490 \text{ cm}^3 * 235 \text{ N/mm}^2 = 115,2 \text{ kNm}$$

4.2 CHS 1000x12

$$R = (100 \text{ cm} - 1,2 \text{ cm})/2 = 49,4 \text{ cm}$$

$$I = \pi * (49,4 \text{ cm})^3 * 1,2 \text{ cm} = 454500 \text{ cm}^4$$

(80 times bigger than 240x12)

$$W = 454500 \text{ cm}^4 / 49,4 \text{ cm} = 9200 \text{ cm}^3$$

(19 times bigger than 240x12)

$$A = 2\pi * 49,4 \text{ cm} * 1,2 \text{ cm} = 372 \text{ cm}^2$$

$$N_{R,k} = 372 \text{ cm}^2 * 235 \text{ N/mm}^2 = 8742 \text{ kN}$$

$$M_{gr,el} = 9200 \text{ cm}^3 * 235 \text{ N/mm}^2 = 2162 \text{ kNm}$$

5 Performance

5.1 Run 1 – 24.02.2017

SIZELE = 1; EFAK = 1

4+6+4 = 14 Elements

NINC = 200

T = 2 sec; step function

258 load steps; 6300 cumulative iterations

CP = 360 sec; I/O = 21:45:28 to 21:49:12 ... app 4 minutes

5.2 Run 2 – 24.02.2017

see run 1, but smaller elements

SIZELE = 1; EFAK = 5

20+30+20 = 70 Elements

NINC = 200

T = 2 sec; step function

258 load steps; 10700 cumulative iterations

CP = 960 sec; I/O = 21:59:09 to 22:05:10 ... app 6 minutes

5.3 Run 3 – 24.02.2017

see run 2, but elastic material

SIZELE = 1; EFAK = 5

20+30+20 = 70 Elements

NINC = 200

T = 2 sec; step function

258 load steps; 258 cumulative iterations

CP = 960 sec; I/O = 22:12:41 to 22:13:15 ... app 0,5 minutes

5.4 Run 4 – 24.02.2017

see run 3, but plastic material and harmonic drive

SIZELE = 1; EFAK = 5

20+30+20 = 70 Elements

NINC = 200

T = 2 sec; harmonic, driving period = $1,635 \text{ s} * 0,95 = 1,553 \text{ s}$

258 load steps; 8025 cumulative iterations

CP = 1437 sec; I/O = 22:17:55 to 22:22:51 ... app 4 minutes

max eppl = 0,72 %

5.5 Run 5 – 24.02.2017

see run 4, but smaller elements

SIZELE = 1; EFAK = 10

40+60+40 = 140 Elements

NINC = 200

T = 2 sec; harmonic, driving period = $1,635 \text{ s} * 0,95 = 1,553 \text{ s}$

258 load steps; 11150 cumulative iterations

CP = 2088 sec; I/O = 22:29:49 to 22:36:44 ... app 7 minutes

remark: CPs are cumulated within one session over all runs

max eppl = 0,97 %

5.6 Run 6 – 25.02.2017

see run 5, but longer duration and changed driving period

SIZELE = 1; EFAK = 10

40+60+40 = 140 Elements

NINC = 200

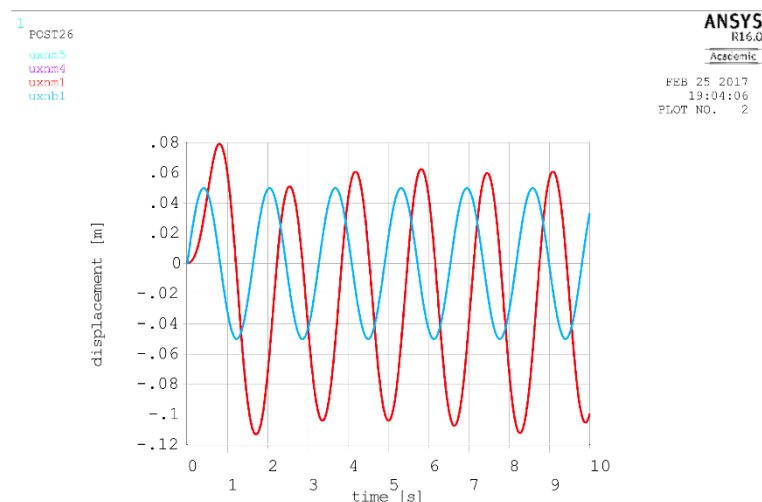
T = 10 sec; harmonic, driving period = 1,635 s

1224 load steps; 65000 cumulative iterations

CP = 3553 sec; I/O = 17:34:45 to 18:10:14 ... app 36 minutes

remark: CPs are cumulated within one session over all runs

max eppl = -1,14 %; +0,46 %



seismic frame w/h m = 6/4 for CompDyn 2017

filename: Run_6_P-f_17-02-25.tif

Max amplitudes of +1./-1 peak:

+79,3 mm (0,801 s)

-113,1 mm (1,709 s)

range = 79,3 mm + 113,1 mm = 192,4 mm

+60,9 mm (9,091 s) quasi-stationary

−105,4 mm (9,900 s) quasi-stationary

(extract from Loesung.txt)

range = 60,9 mm + 113,1 mm = 174,0 mm

5.7 Run 7 – 25.02.2017

see run 6, but elastic with step function to check on the eigenperiod

SIZELE = 1; EFAK = 10

40+60+40 = 140 Elements

NINC = 200

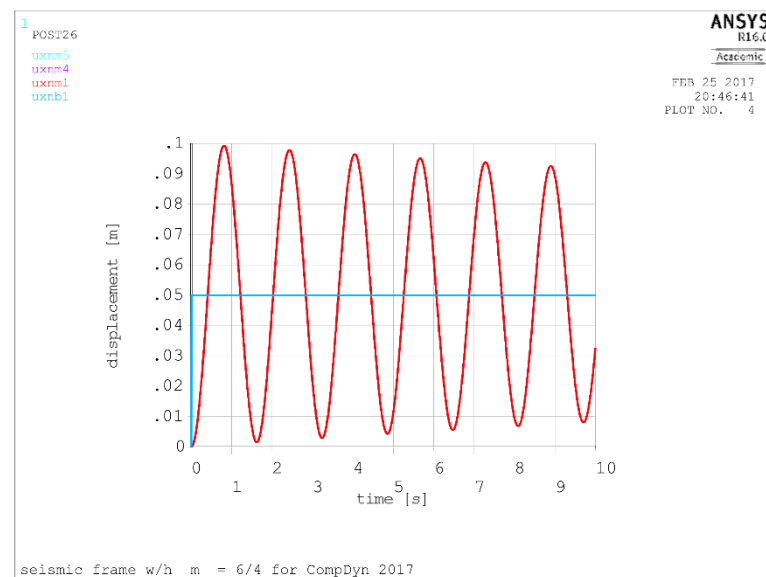
T = 10 sec; step

1224 load steps; 1224 cumulative iterations

CP = 3748 sec; I/O = 20:40:25 to 20:42:50 ... app 2,5 minutes

remark: CPs are cumulated within one session over all runs

max eppl = 0 %



filename: Run_7_P-f_17-02-25.tif

Response of the structure: max/min Amp =

+99,3 mm (0,8093 s)

+1,44 mm (1,6187 s)

+92,6 mm (8,8944 s)

+8,03 mm (9,7037 s)

(extract from Loesung.txt)

$$(8,8944 \text{ s} - 0,8093 \text{ s}) / 5 = 1,6170 \text{ s}$$

$$(9,7037 \text{ s} - 1,6187 \text{ s}) / 5 = 1,6170 \text{ s}$$

Logarithmic decrement is evaluated by means of the Mensinger-formula

$$\delta = 1/n * \ln [\text{Amp}(i) / \text{Amp} (i+n)]$$

$$\delta = 1/5 * \ln [(99,3 \text{ mm} - 50 \text{ mm}) / (92,6 \text{ mm} - 50 \text{ mm})] = 0,0292$$

$$\delta = 1/5 * \ln [(50 \text{ mm} - 1,44 \text{ mm}) / (50 \text{ mm} - 8,03 \text{ mm})] = 0,0290$$

5.8 Run 8 – 25.02.2017

Test of elastic storey drift with $H = 57,6 \text{ kN}$

$$u = 53,0 \text{ mm}$$

should be 52,4 mm; rel err = +1,15 % ... can be attributed to the flexibility of the waler

$$\sigma_{\max} = +0,240 * 10^9 \text{ Pa} = 240 \text{ MPa}$$

$$\sigma_{\min} = -0,240 * 10^9 \text{ Pa} = 240 \text{ MPa}$$

should be 235 MPa; rel err = +2,1 % ... can be attributed to the coarse elements (?)

5.9 Run 9 – 27.02.2017

Elastic time history with period 1,6170 s

$$\text{SIZELE} = 1; \text{EFAK} = 10$$

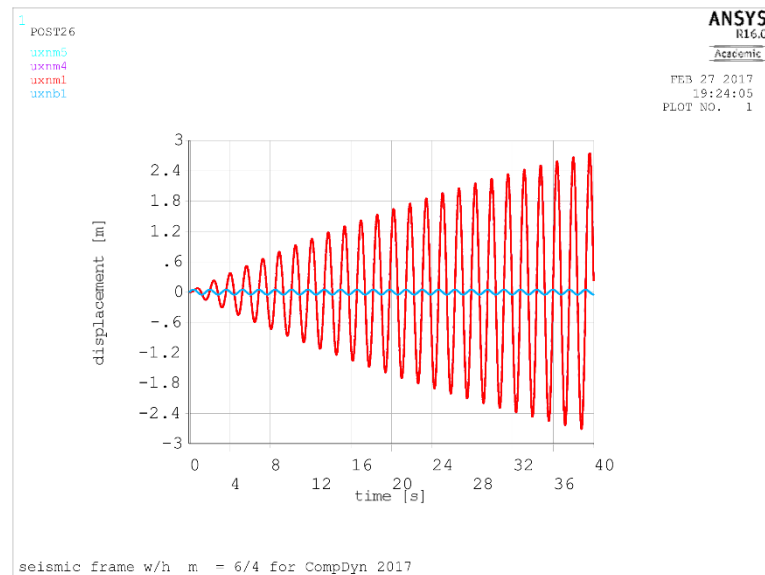
$$40+60+40 = 140 \text{ Elements}$$

$$\text{NINC} = 200$$

$$T = 40 \text{ sec; harmonic}$$

4947 load steps; 4947 cumulative iterations

CP = 846 sec; I/O = 19:07:23 to 19:18:28 ... app 11 minutes



filename: Run_9_P-f_17-02-27.tif

Max amplitudes of +25./-24. peak:

+2.749 mm (39,6165 s)

-2.710 mm (38,8080 s)

For info 1. and 2. peaks

+77,95 mm (0,8085 s)

-154,8 mm (1,6170 s)

+230,5 mm (2,4255 s)

-305,1 mm (3,234 s)

5.10 Run 10 – 27.02.2017

see run 9 but plastic

SIZELE = 1; EFAK = 10

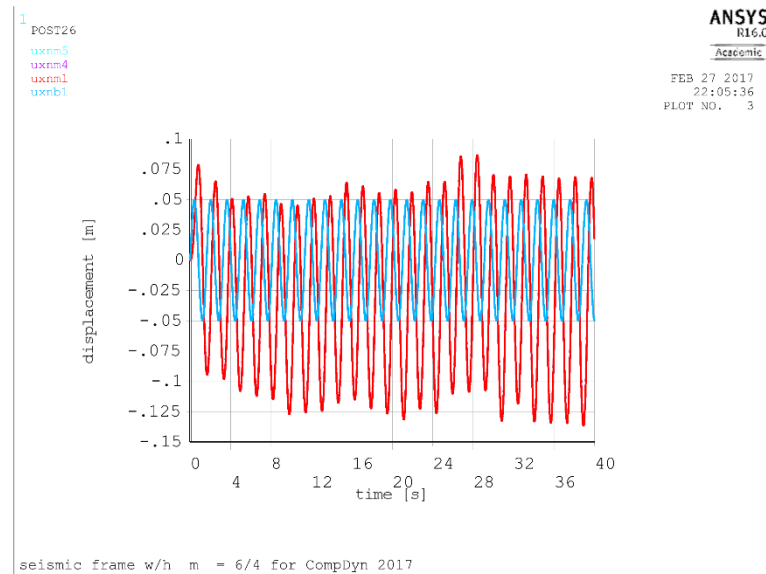
40+60+40 = 140 Elements

NINC = 200

T = 40 sec; harmonic

4948 load steps; 280007 cumulative iterations

CP = 12703 sec; I/O = 19:53:38 to 21:55:16 ... app 122 minutes



filename: Run_10_P-f_17-02-27.tif

Max amplitudes of +18. peak:

+86,9 mm (28,4026 s)

−108,7 mm (27,5941 s) (previous negative peak)

range = 86,9 mm + 108,7 mm = 195,6 mm

Max amplitudes of −24. peak:

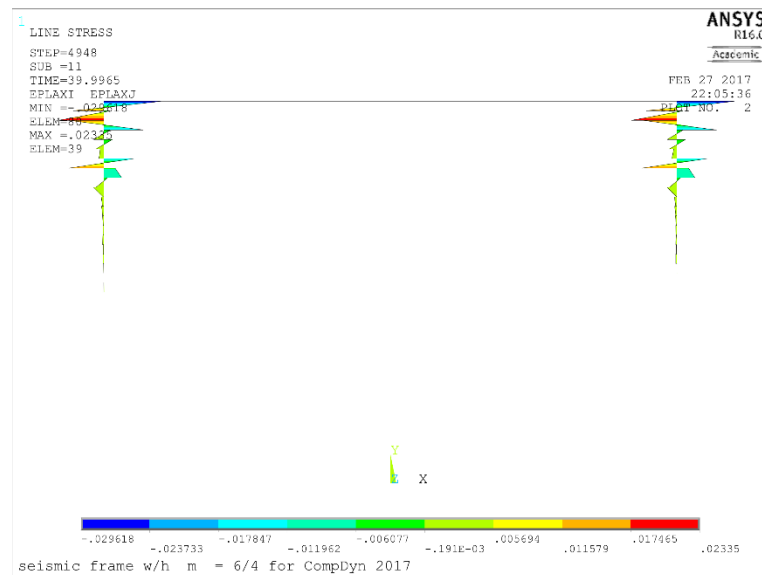
−136,5 mm (38,9131 s)

+68,8 mm (38,1046 s) (previous positive peak)

range = 68,8 mm + 136,5 mm = 205,3 mm

For info 1. peak

+78,9 mm (0,7923 s)



filename: Run_10_eppl_17-02-27.tif

5.11 Run 11 – 28.02.2017

see run 10 but driving amplitude ± 25 mm instead of 50 mm.

SIZELE = 1; EFAK = 10

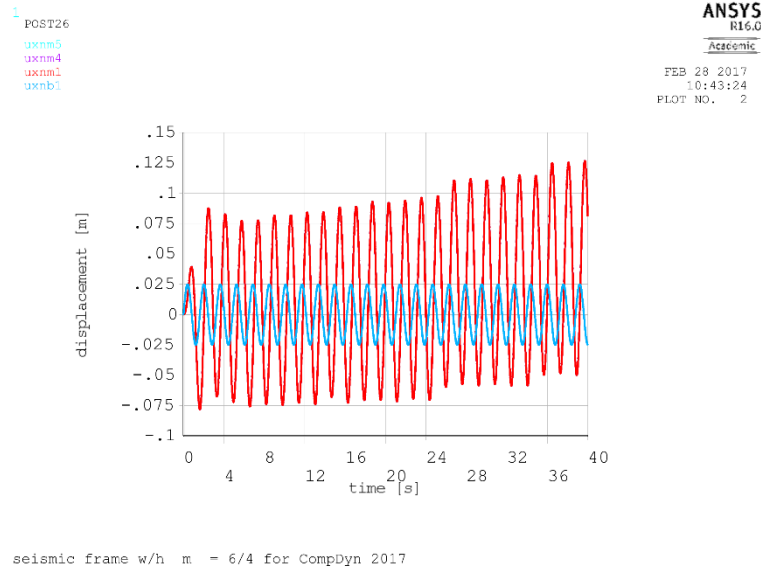
40+60+40 = 140 Elements

NINC = 200

T = 40 sec; harmonic

4948 load steps; 198682 cumulative iterations

CP = 2531 to 11299 sec; I/O = 07:53:45 to 09:49:31 ... app 116 minutes



filename: Run_11_P-f_17-02-28.tif

Max amplitudes of +25. peak:

+126,8 mm (39,7216 s)

−50,0 mm (38,9131 s) (previous negative peak)

range = 126,8 mm + 50,0 mm = 176,8 mm

Max amplitudes of −2. peak:

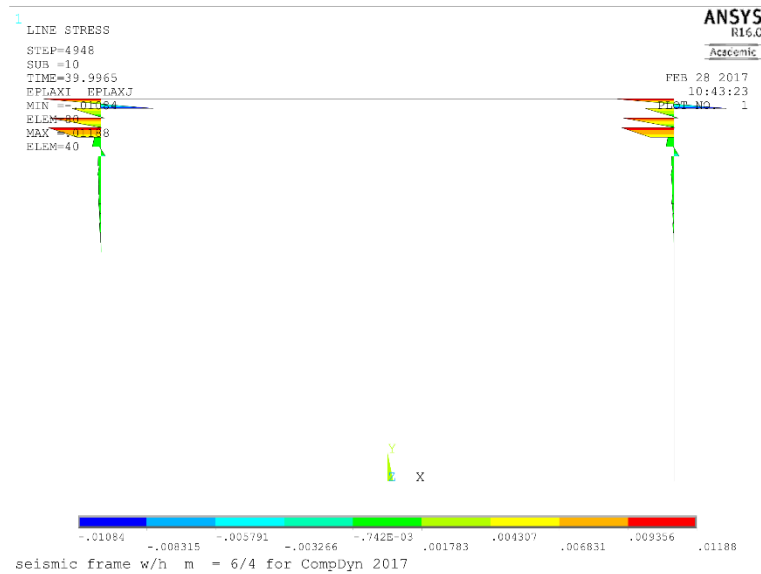
−78,2 mm (1,6089 s)

+87,8 mm (2,4740 s) (following positive peak)

range = 78,2 mm + 87,8 mm = 166,0 mm

For info 1. peak

+39,4 mm (0,7923 s)



filename: Run_11_eppl_17-02-28.tif

5.12 Run 12 – 28.02.2017

see run 10 but driving amplitude ± 100 mm instead of 50 mm (see also run 11)

SIZELE = 1; EFAK = 10

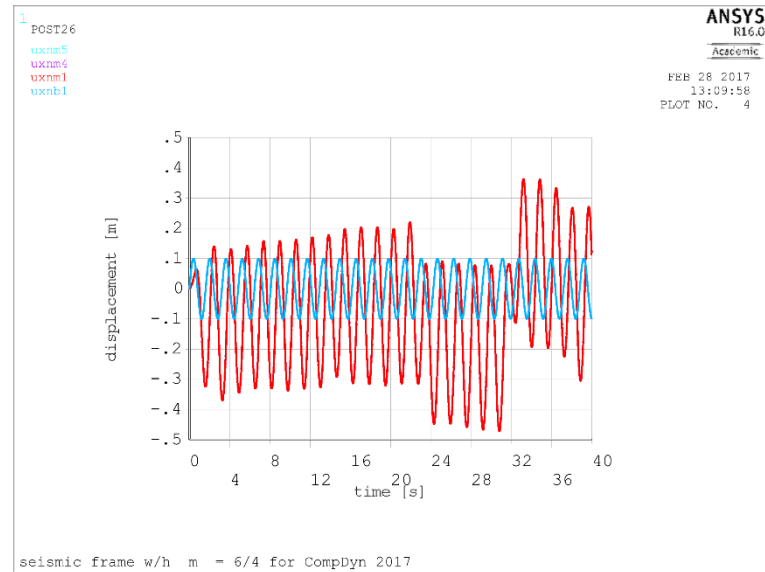
40+60+40 = 140 Elements

NINC = 200

T = 40 sec; harmonic

4948 load steps; 277087 cumulative iterations

CP = 11301 to 23677 sec; I/O = 10:52:38 to 12:57:31 app 125 minutes



filename: Run_12_P-f_17-02-28.tif

Max amplitudes of +21. peak:

+363,4 mm (33,2132 s)

−193,1 mm (34,0298 s) (following negative peak)

range = 363,4 mm + 193,1 mm = 556,5 mm

Max amplitudes of −19. peak:

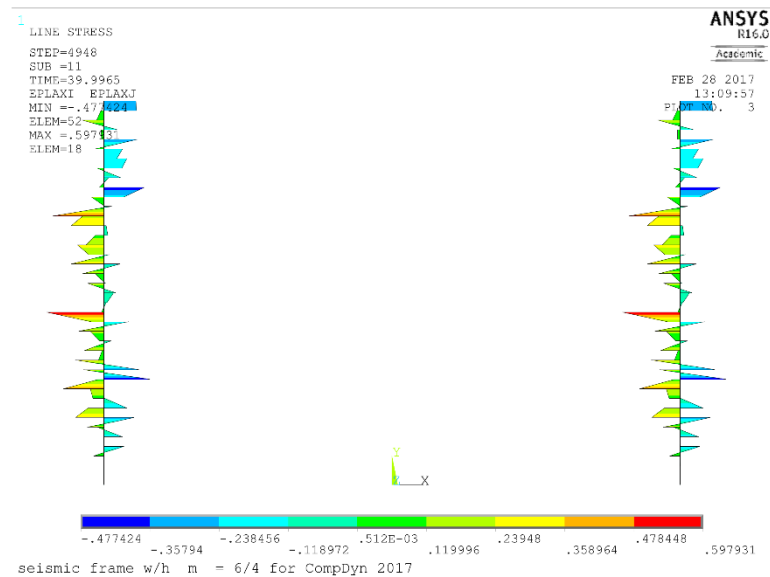
−472,2 mm (30,8200 s)

+82,2 mm (31,7579 s) (following positive peak)

range = 472,2 mm + 82,2 mm = 554,4 mm

For info 1. peak

+64,2 mm (0,7034 s)



filename: Run_12_eppl_17-02-28.tif

5.13 Run 13 – 28.02.2017

elastic with step function to check on the eigen period, see run 7, but masses reduced by a factor of two

SIZELE = 1; EFAK = 10

40+60+40 = 140 Elements

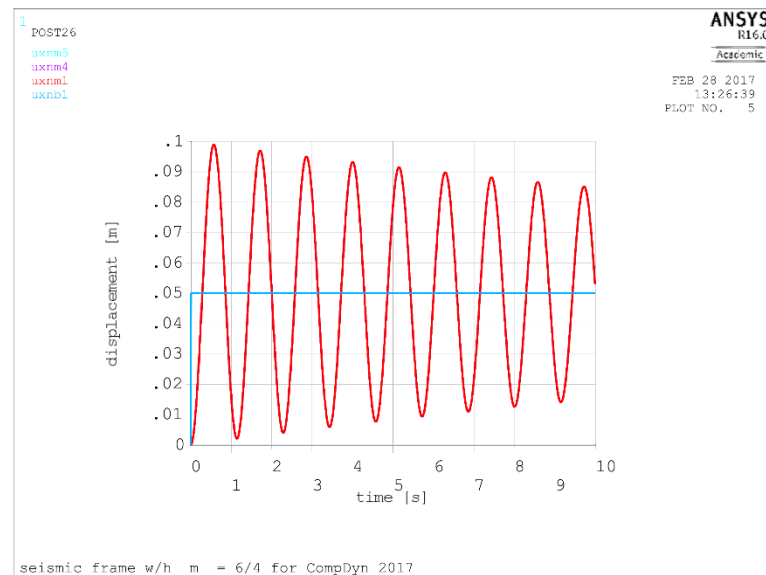
NINC = 200

T = 10 sec; step

1237 load steps; 1237 cumulative iterations

CP = 23678 to 23885 sec; I/O = 13:20:19 to 13:22:58 ... app 2,5 minutes

max eppl = 0 %



filename: Run_13_P-f_17-02-28.tif

Response of the structure: max/min Amp =

+0,0 mm (0,0 s)

+99,0 mm (0,5740 s)

+2,0 mm (1,1481 s)

+97,0 mm (1,7140 s)

+14,1 mm (9,1522 s)

+85,2 mm (9,7182 s)

(extract from Loesung.txt)

$$(9,7182 \text{ s} - 0,5740 \text{ s}) / 8 = 1,1430 \text{ s}$$

$$(9,1522 \text{ s} - 1,1481 \text{ s}) / 7 = 1,1434 \text{ s}$$

Logarithmic decrement is evaluated by means of the Mensinger-formula

$$\delta = 1/n * \ln [\text{Amp}(i) / \text{Amp}(i+n)]$$

$$\delta = 1/8 * \ln [(99,0 \text{ mm} - 50 \text{ mm}) / (85,2 \text{ mm} - 50 \text{ mm})] = 0,0413$$

$$\delta = 1/7 * \ln [(50 \text{ mm} - 2,0 \text{ mm}) / (50 \text{ mm} - 14,1 \text{ mm})] = 0,0415$$

5.14 Run 14 – 28.02.2017

Elastic time history with period 1,1430 s, see Run_9

SIZELE = 1; EFAK = 10

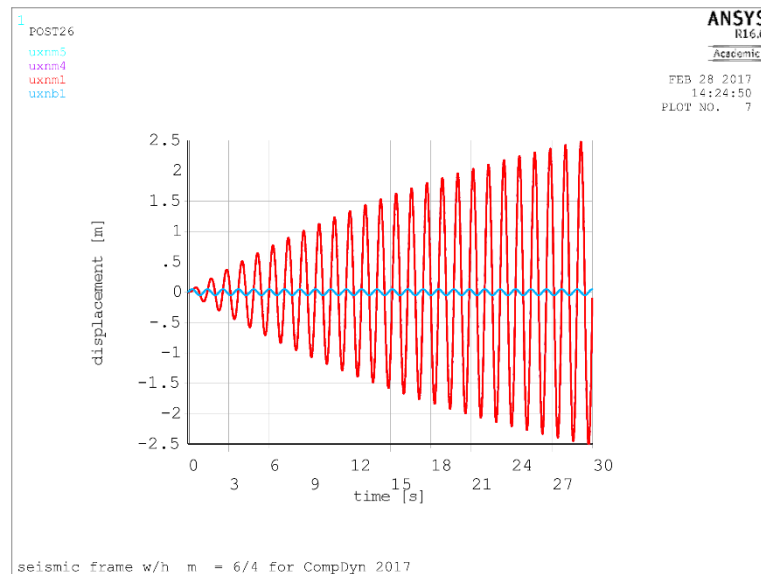
40+60+40 = 140 Elements

NINC = 200

T = 30 sec; harmonic

5250 load steps; 5250 cumulative iterations

CP = 23886 to 24866 sec; I/O = 13:57:12 to 14:09:01 ... app 12 minutes



filename: Run_14_P-f_17-02-28.tif

Max amplitudes of +26./-26. peak:

+2.482 mm (29,1465 s)

-2.509 mm (29,7180 s)

For info 1. and 2. peaks

+77,7 mm (0,5658 s)

-153,8 mm (1,1430 s)

+228,4 mm (1,7145 s)

-301,5 mm (2,2860 s)

5.15 Run 15 – 28.02.2017

see Run_14, but plastic

SIZELE = 1; EFAK = 10

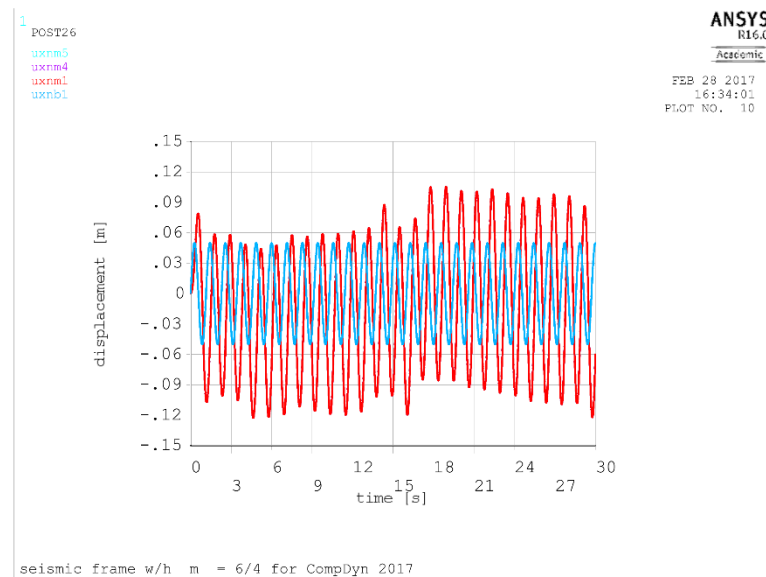
40+60+40 = 140 Elements

NINC = 200

T = 30 sec; harmonic

5250 load steps; 245833 cumulative iterations

CP = 24870 to 36760 sec; I/O = 14:30:19 to 16:28:38 ... app 118 minutes



filename: Run_15_P-f_17-02-28.tif

Max amplitudes of +16. peak:

+87,7 mm (14,3561 s)

-101,8 mm (13,7732 s) (previous negative peak)

range = 87,7 mm + 101,8 mm = 189,5 mm

Max amplitudes of -4. peak:

-121,8 mm (4,6120 s)

+43,6 mm (5,1892 s) (previous positive peak)

range = 121,8 mm + 43,6 mm = 165,4 mm

Max amplitudes of -26. peak:

-122,2 mm (29,7809 s)

+86,6 mm (29,2094 s) (previous positive peak)

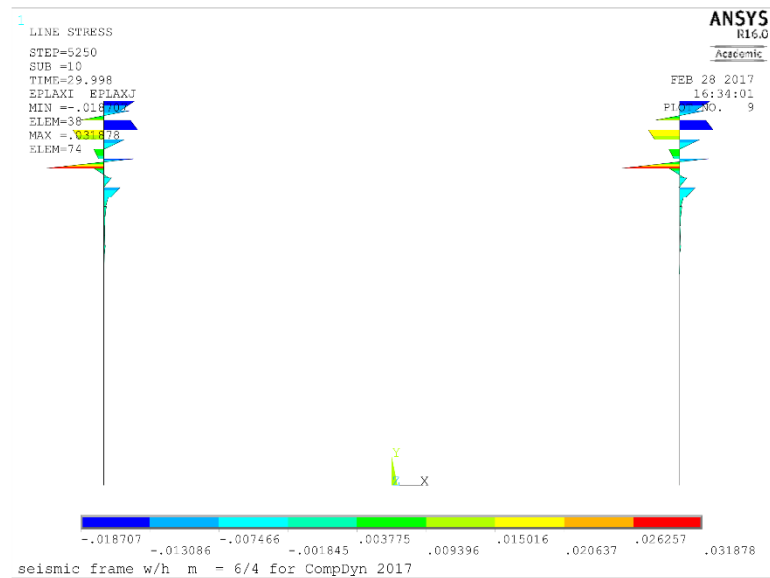
range = 122,2 mm + 86,6 mm = 208,8 mm

For info +1./-1. peaks

+79,1 mm (0,5544 s)

-107,3 mm (1,1830 s)

range = 79,1 mm + 107,3 mm = 186,4 mm



filename: Run_15_eppl_17-02-28.tif

5.16 Run 16 – 28.02.2017

see run 15 but driving amplitude ± 25 mm instead of 50 mm.

SIZELE = 1; EFAK = 10

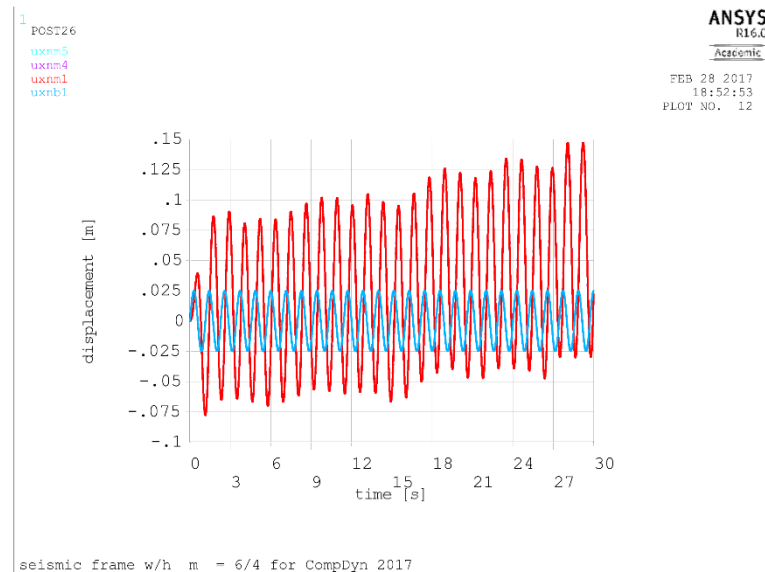
40+60+40 = 140 Elements

NINC = 200

T = 30 sec; harmonic

5250 load steps; 176010 cumulative iterations

CP = 36761 to 46158 sec; I/O = 16:43:16 to 18:15:44 ... app 92 minutes



filename: Run_16_P-f_17-02-28

Max amplitudes of -1. peak:

-77,9 mm (1,1373 s)

+86,6 mm (1,7374 s) (following positive peak)

range = 77,9 mm + 86,6 mm = 164,5 mm

Max amplitudes of +26. peak:

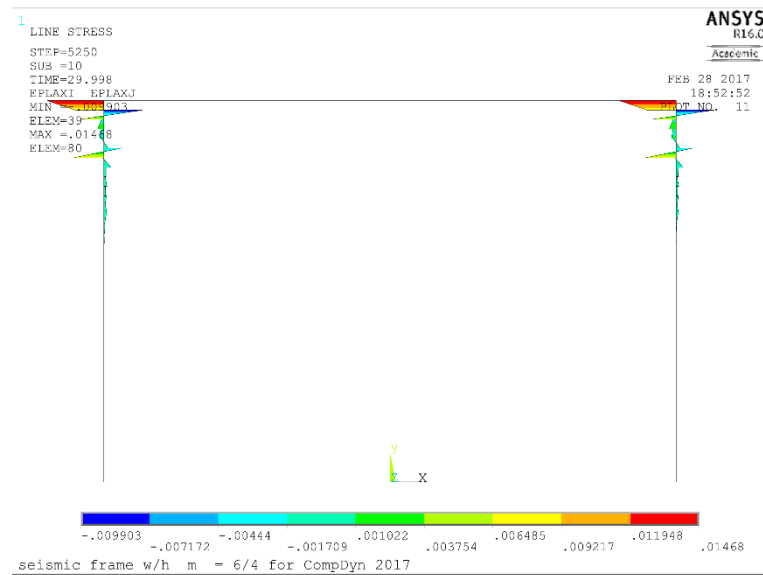
+147,3 mm (29,2151 s)

-30,2 mm (28,6436 s) (previous negative peak)

range = 147,3 mm + 30,2 mm = 177,5 mm

For info +1. peak

+39,5 mm (0,5544 s)



5.17 Run 17 – 28.02.2017

see run 15 and 16 but driving amplitude ± 100 mm instead of 50 mm.

SIZELE = 1; EFAK = 10

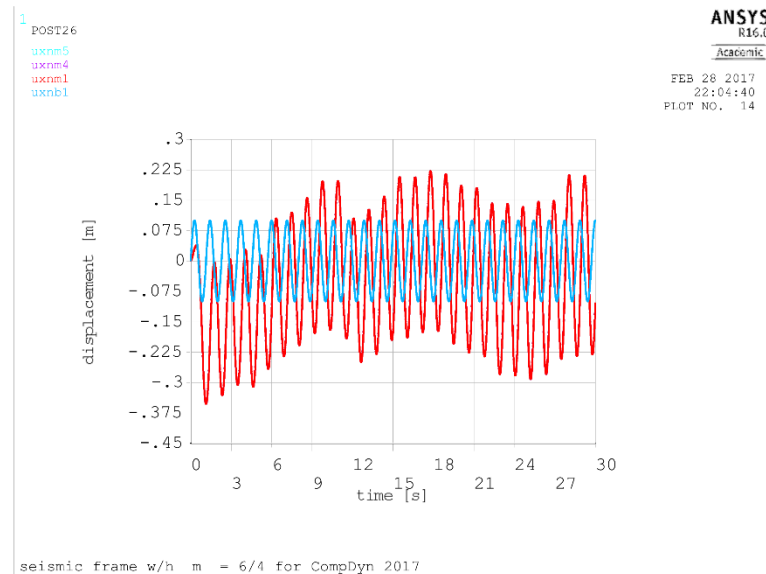
40+60+40 = 140 Elements

NINC = 200

T = 30 sec; harmonic

5250 load steps; 252464 cumulative iterations

CP = 46160 to 61165 sec; I/O = 19:22:56 to 21:49:44 ... app 147 minutes



filename: Run_17_P-f_17-02-28.tif

Max amplitudes of +16. peak:

+222,2 mm (17,7794 s)

−176,1 mm (17,2079 s) (previous negative peak)

range = 222,2 mm + 176,1 mm = 398,3 mm

Max amplitudes of −1. peak:

−352,8 mm (1,1430 s)

+38,6 mm (0,4172 s) (previous positive peak)

range = 352,8 mm + 38,6 mm = 391,4 mm

Max amplitudes of +25. peak:

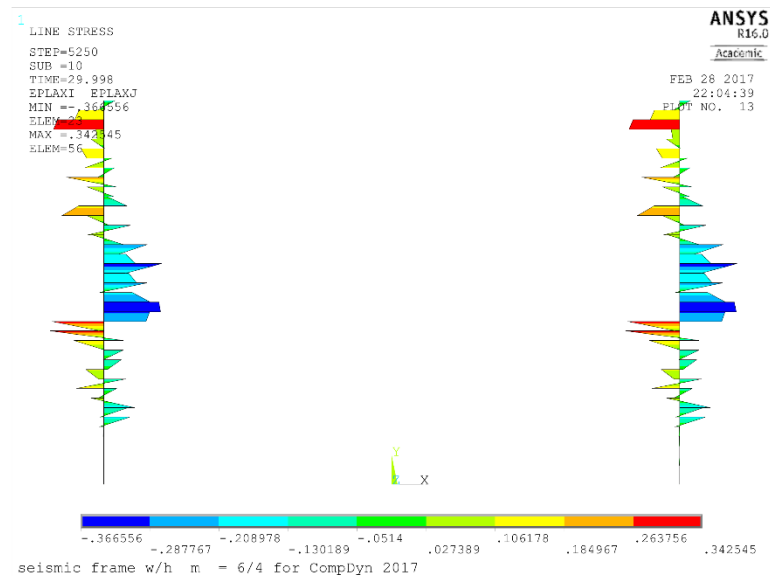
+212,0 mm (28,0607 s)

−242,2 mm (27,4834 s) (previous negative peak)

range = 212,0 mm + 242,2 mm = 454,2 mm

For info +1. peak

+38,6 mm (0,4172 s)



filename: Run_17_eppl_17-02-28.tif

5.18 Run 18 – 28.02.2017

elastic with step function to check on the eigen period, see run 7, but masses doubled

SIZELE = 1; EFAK = 10

40+60+40 = 140 Elements

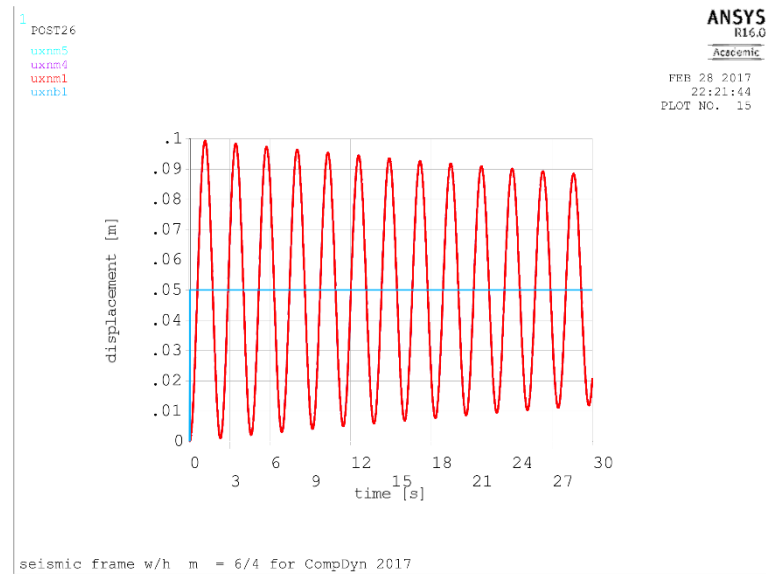
NINC = 200

T = 30 sec; step

2490 load steps; 2490 cumulative iterations

CP = 61168 to 61618 sec; I/O = 22:12:24 to 22:18:20 ... app 6 minutes

max eppl = 0 %



filename: Run_18_P-f_17-02-28.tif

max amplitude of +1. peak (extract from Loesung.txt)

+99,4 mm (1,1446 s)

max amplitude of +13. peak

+88,5 mm (28,5911 s)

max amplitude of –1. peak

+1,0 mm (2,2892 s)

max amplitude of –13. peak

+11,9 mm (29,7357 s)

$$(28,5911 \text{ s} - 1,1446 \text{ s}) / 12 = 2,2872 \text{ s}$$

$$(29,7357 \text{ s} - 2,2892 \text{ s}) / 12 = 2,2872 \text{ s}$$

Logarithmic decrement is evaluated by means of the Mensinger-formula

$$\delta = 1/n * \ln [\text{Amp}(i) / \text{Amp} (i+n)]$$

$$\delta = 1/12 * \ln [(99,4 \text{ mm} - 50 \text{ mm}) / (88,5 \text{ mm} - 50 \text{ mm})] = 0,0208$$

$$\delta = 1/12 * \ln [(50 \text{ mm} - 1,0 \text{ mm}) / (50 \text{ mm} - 11,9 \text{ mm})] = 0,0210$$

5.19 Run 19 – 28.02.2017

Elastic time history with period 2,2872 s, see Run_18

SIZELE = 1; EFAK = 10

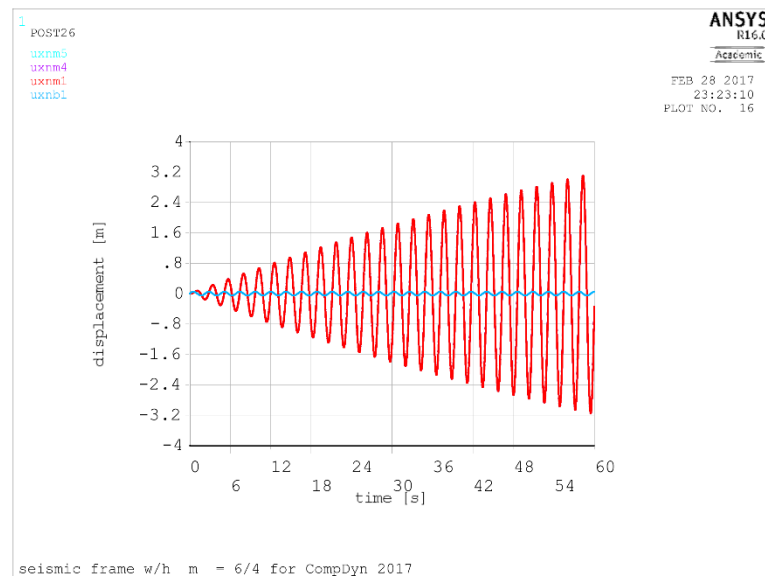
40+60+40 = 140 Elements

NINC = 200

T = 60 sec; harmonic

5247 load steps; 5247 cumulative iterations

CP = 62640 to 63687 sec; I/O = 23:08:23 to 23:21:49 ... app 13 minutes



filename: Run_19_P-f_17-02-28.tif

Max amplitudes of +26./-26. peak:

+3.109 mm (58,3122 s)

-3.155 mm (59,4558 s)

For info 1. and 2. peaks

+78,1 mm (1,1436 s)

-155,4 mm (2,2872 s)

+232,0 mm (3,4308 s)

-307,7 mm (4,5744 s)

5.20 Run 20 – 28.02.2017

as Run_19 but plastic

SIZELE = 1; EFAK = 10

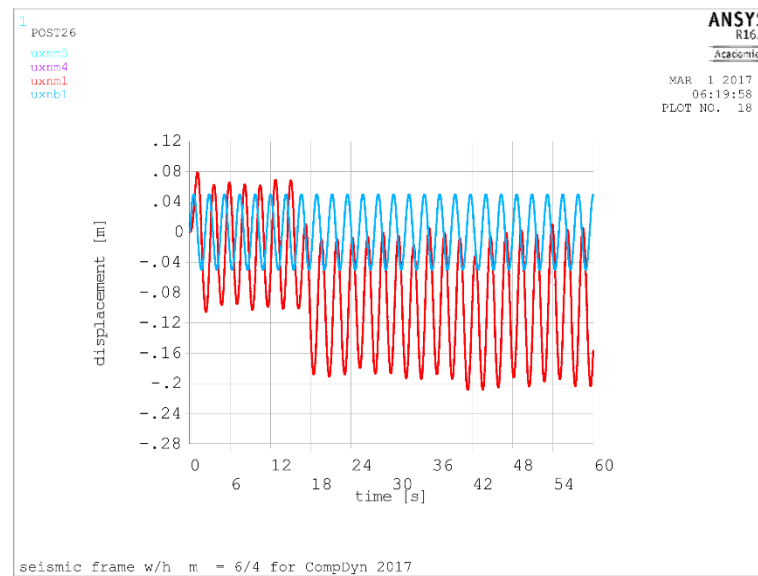
40+60+40 = 140 Elements

NINC = 200

T = 60 sec; harmonic

5247 load steps; 314829 cumulative iterations

CP = 63688 to 81563 sec; I/O = 23:30:06 to 02:23:12 ... app 173 minutes



filename: Run_20_P-f_17-02-28.tif

Max amplitudes of +1./-1. peak:

+78,9 mm (1,1322 s)

-103,8 mm (2,3101 s)

range = 78,9 mm + 103,8 mm = 182,7 mm

Max amplitudes of -18. peak:

-208,3 mm (41,3411 s)

-8,2 mm (40,1632 s) (previous positive peak)

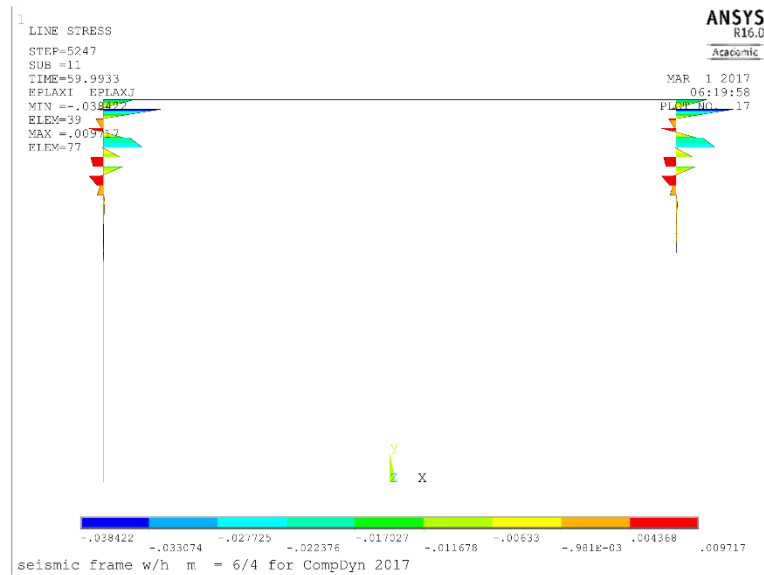
range = 208,3 mm - 8,2 mm = 200,1 mm

Max amplitudes of +26./-26. peak:

+5,3 mm (58,4723 s)

-203,7 mm (59,6159 s) (following negative peak)

range = 5,3 mm + 203,7 mm = 209,0 mm



filename: Run_20_eppl_17-02-28.tif

5.21 Run 21 – 01.03.2017

as Run_20 but driving amplitude ± 100 mm

SIZELE = 1; EFAK = 10

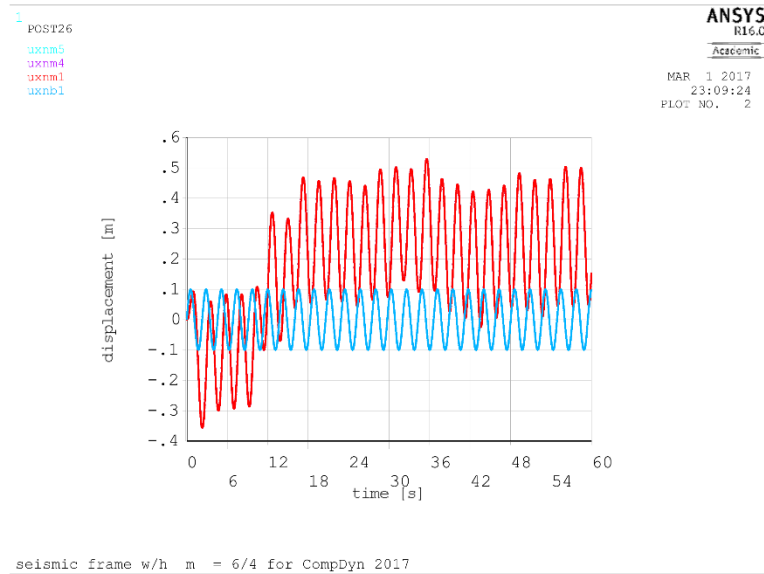
40+60+40 = 140 Elements

NINC = 200

T = 60 sec; harmonic

5247 load steps; 339534 cumulative iterations

CP = 2 to 19444 sec; I/O = 19:56:45 to 23:06:21 ... app 190 minutes



filename: Run_21_P-f_17-03-01.tif

Max amplitudes of +1./-1. peak:

+93,5 mm (0,9606 s) (previous positive peak)

-356,3 mm (2,2986 s)

range = 93,5 mm + 356,3 mm = 449,8 mm

Max amplitudes of +16. peak:

+529,4 mm (35,5774 s)

+9,1 mm (34,4109 s) (previous negative peak)

range = 529,4 mm - 9,1 mm = 520,3 mm

Max amplitudes of -19. peak:

-24,8 mm (43,5940 s)

+428,2 mm (44,7719 s) (following positive peak)

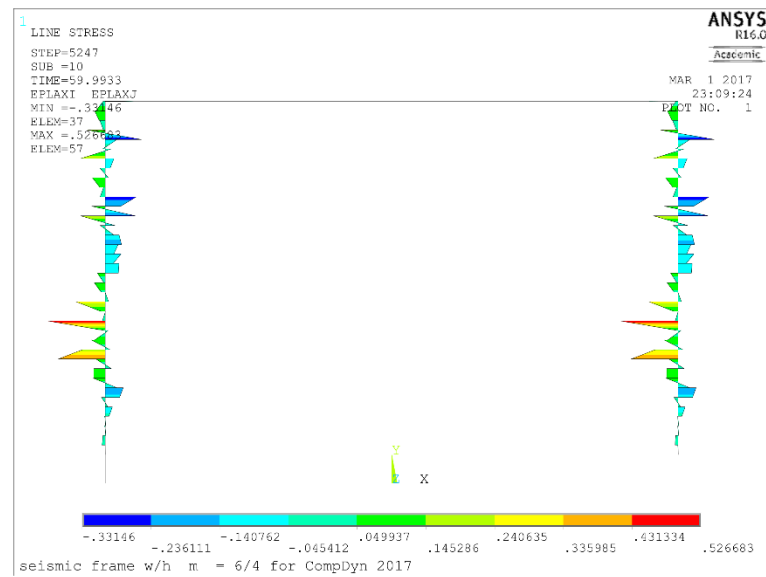
range = 24,8 mm + 428,2 mm = 453,0 mm

Max amplitudes of +26./-26. peak:

+500,4 mm (58,4608 s)

+45,9 mm (59,6159 s) (following negative peak)

range = 500,4 mm - 45,9 mm = 454,5 mm



filename: Run_21_eppl_17-03-01.tif

5.22 Run 22 – 01.03.2017

as Run_21 but driving amplitude ± 25 mm

SIZELE = 1; EFAK = 10

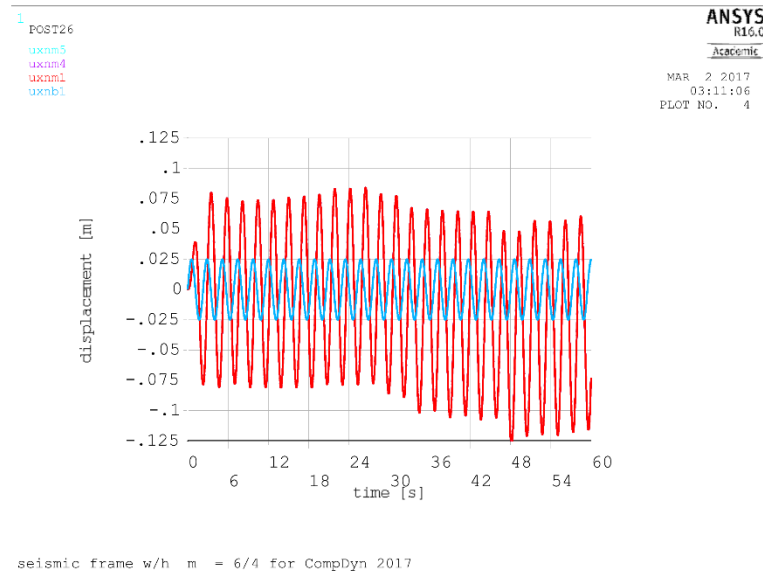
40+60+40 = 140 Elements

NINC = 200

T = 60 sec; harmonic

5247 load steps; 248081 cumulative iterations

CP = 19446 to 30459 sec; I/O = 23:16:04 to 01:06:26 ... app 110 minutes



filename: Run_22_P-f_17-03-01.tif

Max amplitudes of +12. peak:

+84,2 mm (26,4629 s)

−78,5 mm (25,3193 s) (previous negative peak)

range = 84,2 mm + 78,5 mm = 162,7 mm

Max amplitudes of −21. peak:

−128,6 mm (48,1684 s)

+48,5 mm (47,0020 s) (previous positive peak)

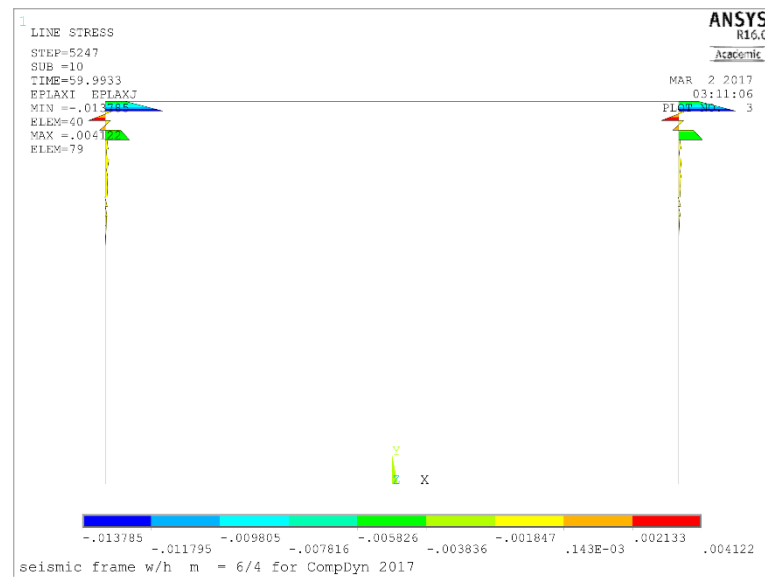
range = 128,6 mm + 48,5 mm = 177,1 mm

Max amplitudes of +26./−26. peak:

+60,5 mm (58,4723 s)

−115,9 mm (59,6159 s) (following negative peak)

range = 60,5 mm + 115,9 mm = 176,4 mm



filename: Run_22_eppl_17-03-01.tif

5.23 Run 23 – 03.03.2017 – push-over

Prescribed load $57.6 \cdot 1.5 = 86,4$ kN

SIZELE = 1; EFAK = 10

40+60+40 = 140 Elements

NSUB,100,10000,100

158 sub steps; 1672 cumulative iterations

CP = 68 to 168 sec; I/O = 11:45:21 to 11:46:25 ... app 1 minute

Last converged at $0.827600 \cdot 86,4$ kN = 71,50 kN

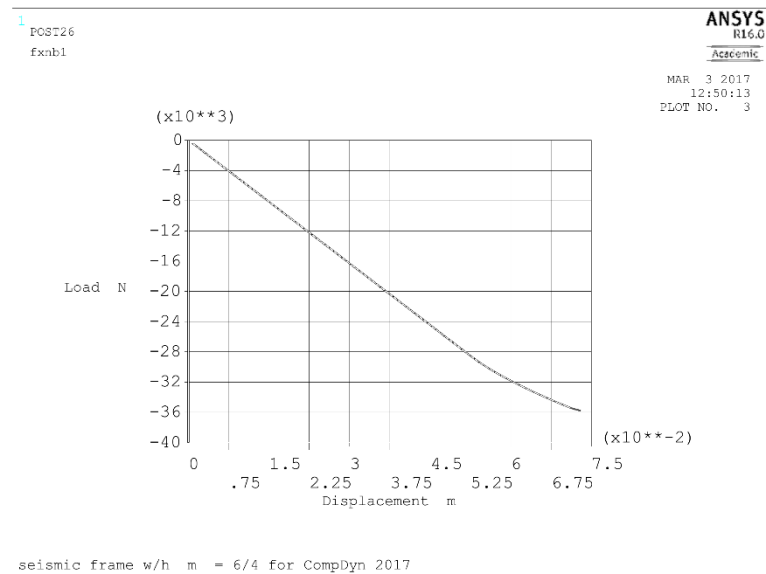
Corresponds to $71,5$ kN / $73,1$ kN = 0,98 of the theoretical plastic load.

Remark 1: the elastic limit load of the model has been shown to be at –2 % as well.

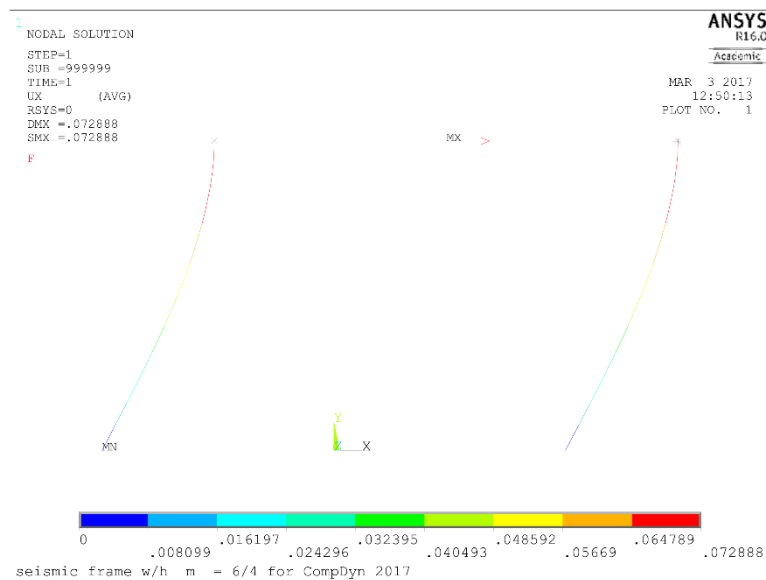
Remark 2: Since material hardening is used, there should not be a plastic limit load

max eppl = 0,120 %

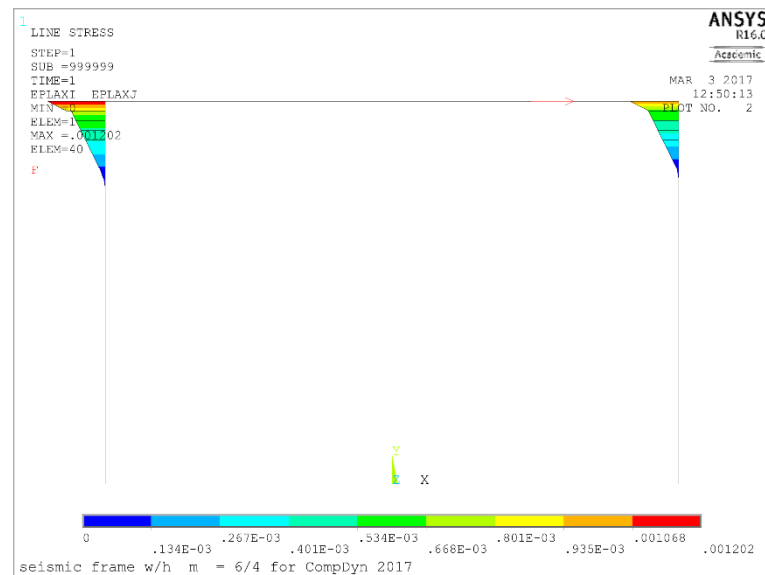
may ux = 72,9 mm



filename: Run_23_P-f_17-03-03.tif



filename: Run_23_ux_17-03-03.tif



filename: Run_23_eppl_17-03-03.tif

5.24 Run 24 – 03.03.2017 – push-over

see Run_24 but quadruple number of elements

SIZELE = 1; EFAK = 40

160+240+160 = 560 Elements

NSUB,100,10000,100

158 sub steps; 1672 cumulative iterations

CP = 179 to 247 sec; I/O = 12:34:39 to 12:35:18 ... app 1 minute

Last converged at $0.82423 \cdot 86,4 \text{ kN} = 71,21 \text{ kN}$

Corresponds to $71,2 \text{ kN} / 73,1 \text{ kN} = 0,974$ of the theoretical plastic load.

max eppl = 0,113 %

may ux = 72,0 mm

5.25 Run 25 – 03.03.2017 – push-over

see Run_23 – double mass with gravity and second order theory

SIZELE = 1; EFAK = 10

40+60+40 = 140 Elements

NSUB,100,10000,100

145 sub steps; 2122 cumulative iterations

CP = 2 to 208 sec; I/O = 16:13:37 to 16:19:34 ... app 6 minutes

Last converged at $0.58512 * 86,4 \text{ kN} = 50,55 \text{ kN}$

max eppl = 0,0853 %

may ux = 78,9 mm

Support reactions: (node; x; y)

1; -26,660 kN; +368,78 kN

2; -23,903 kN; +457,92 kN

Sum of horizontal: -50,56 kN

Mean of vertical: +413,35 kN ... should be $72.000 \text{ kgs} * 9,81 \text{ m/s}^2 = 706 \text{ kN}$

Half diff of vertical: +44,57 kN ... should be $50,55 \text{ kN} * 4 \text{ m} / 6 \text{ m} = 33,7 \text{ kN}$

Remark: a test with gravity but without horizontal loads gives a normal force of -706,320 kN in each of the columns – q.e.d.

5.26 Run 26 – 05.03.2017 – push-over

A lot of trial to get out data for a Load-displacement diagram properly.

The above error in the above vertical forces was due to the fact, that gravity was ramped up with the horizontal loads. Now this is fixed by having a preliminary solution step for the gravity, and then subsequently stepwise horizontal loads.

Purely elastic pushover with 2 masses of 72.000 kgs each and gravity.

For results see Run_30.

5.27 Run 27 – 05.03.2017 – push-over

As Run_26 but without gravity on the masses – i.e. vertical force and second order effects

see L-d-path

5.28 Run 28 – 05.03.2017 – push-over

As Run_27 but with plasticity

Prescribed load is $57,6 \text{ kN} * 1,3 = 74,88 \text{ kN}$

SIZELE = 1; EFAK = 40

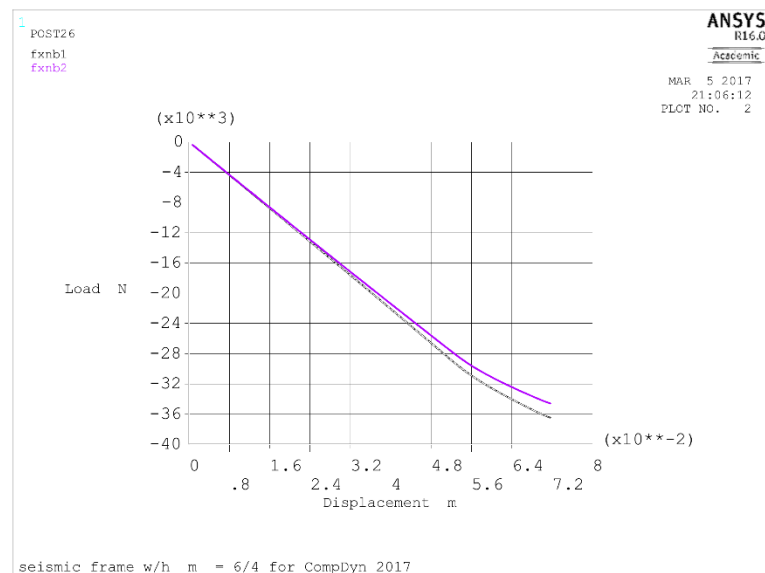
$160+240+160 = 560$ Elements

NSUB,100,10000,100

99 sub steps; 557 cumulative iterations

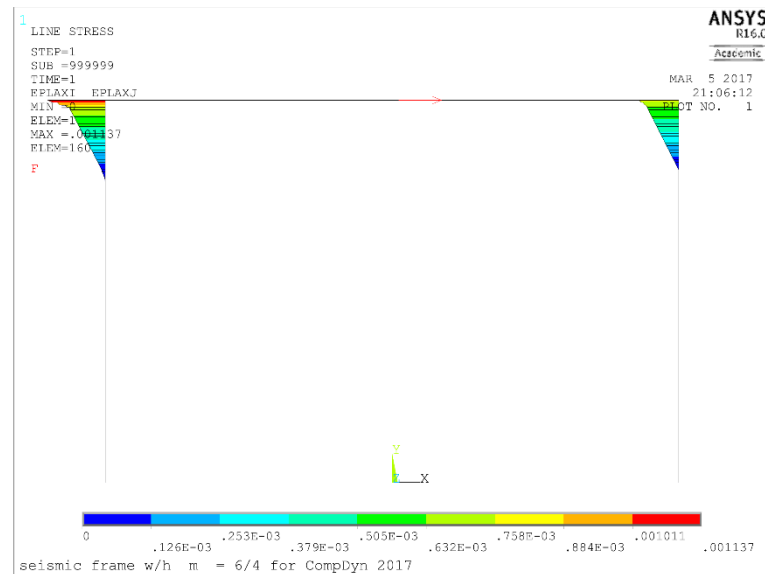
CP = 2 to 89 sec; I/O = 20:56:47 to 20:57:31 ... app 1 minutes

Last converged at $0.94883 * 74,88 \text{ kN} = 71,05 \text{ kN}$



filename: Run_28_P-f_17-03-05.tif

For lack of explicit L-d-data see Run_31.



filename: Run_28_eppl_17-03-05.tif

5.29 Run 29 – 05.03.2017 – push-over

As Run_28 but with gravity

Prescribed load is $57,6 \text{ kN} * 1,3 = 74,88 \text{ kN}$

SIZELE = 1; EFAK = 40

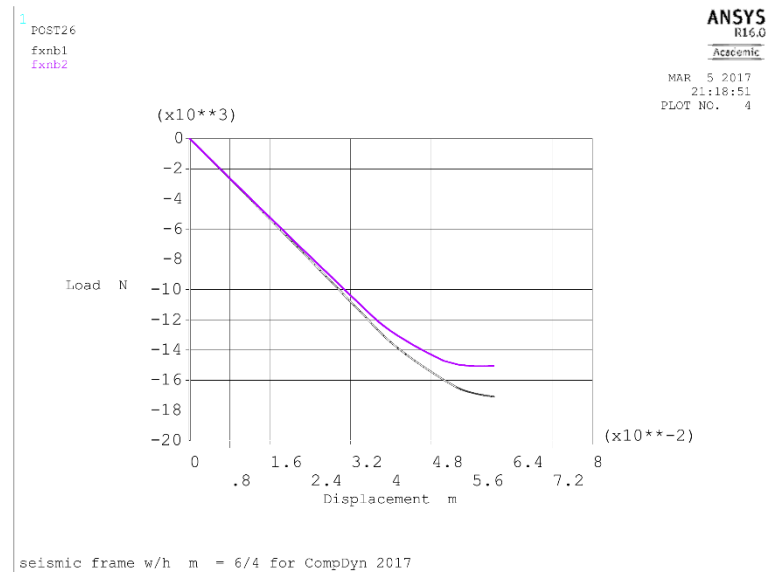
$160+240+160 = 560$ Elements

NSUB,100,10000,100

85 sub steps; 1309 cumulative iterations

CP = 2 to 114 sec; I/O = 22:04:53 to 22:06:10 ... app 1,5 minutes

Last converged at $(1.4291 - 1) * 74,88 \text{ kN} = 32,13 \text{ kN}$



filename: Run_29_P-f_17-03-05.tif

no plastic strains shown in the graph;

however, in the log plastic strains are indicated from $(1.3300 - 1) * 74,88 \text{ kN} = 24,71 \text{ kN}$

5.30 Run 30 – 06.03.2017 – push-over

As Run_29 but without plasticity

Prescribed load is $57,6 \text{ kN} * 3 = 172,80 \text{ kN}$

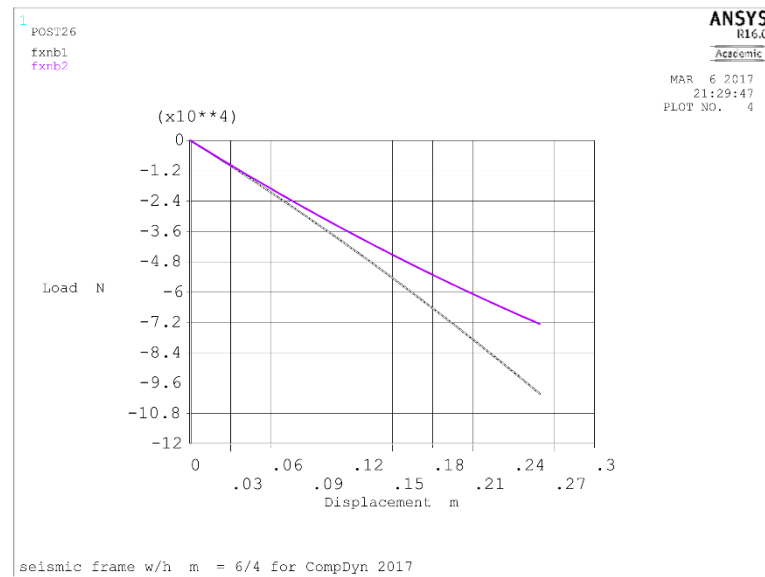
SIZELE = 1; EFAK = 40

$160 + 240 + 160 = 560$ Elements

NSUB,100,10000,100

100 sub steps; 200 cumulative iterations

CP = 137 to 152 sec; I/O = 21:26:45 to 21:26:54 ... app 0,2 minutes



filename: Run_30_P-f_17-03-06.tif

5.31 Run 31 – 12.03.2017 – push-over

Re-Run of Run_28: plasticity – no gravity – only to get L-d-data

Prescribed load is $57,6 \text{ kN} * 1,3 = 74,88 \text{ kN}$

SIZELE = 1; EFAK = 40

$160+240+160 = 560$ Elements

NSUB,100,10000,100

99 sub steps; 557 cumulative iterations

CP = 2 to 55 sec; I/O = 17:56:06 to 17:56:57 ... app 1 minutes

Last converged at $0.94883 * 74,88 \text{ kN} = 71,05 \text{ kN}$

6 Evaluation

Run no.	drive [±mm]	w,max,+ [mm]	w,max,- [mm]	range [mm]	DAF+	DAF-	DAF,r	remark
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6	50	79,3	113,1	192,4	1,59	2,26	1,94	only 5,5 periods
10	50	86,9	136,5	205,3	1,74	2,73	2,05	
11	25	126,8	78,2	176,8	5,07	3,13	3,54	
12	100	363,4	472,2	556,5	3,63	4,72	2,78	
15	50	87,7	122,2	208,8	1,75	2,44	2,09	mass half
16	25	147,3	77,9	177,5	5,89	3,12	3,55	mass half
17	100	222,2	352,8	454,2	2,22	3,53	4,54	mass half
20	50	78,9	208,3	209,0	1,58	4,17	2,09	mass double
21	100	529,4	356,3	520,3	5,29	3,56	2,60	mass double
22	25	84,2	128,6	177,1	3,37	5,14	3,54	mass double

7 Graphix

For some reason saving as pdf-to-tiff gives no satisfying results.

Print pdf with 1200 dpi, P-f-diagramm is 173 kB; clip to desired size;

Open GIMP with option 300 dpi

Export by suffix; TIFF; LZW-Compression; tiff has 497 kB