EuroSteel 2014

Background Document

on FE Analyses

0 Contents

1 General

This document reports on the structural analyses for

Knoedel/Hrabowski/Ummenhofer

Seismic behaviour factor in combined frame and braced structures

to be prepared for

Eurosteel 2014, 7th EUROpean conference on STEEL and Composite Structures, Napoli, 10-12 September 2014

2 Procedure

2.1 Structures

In Knoedel/Hrabowski 2012 a realistic PGA of 2,4 m/s^2 has been chosen. A frame has been designed which under the given seismic loads and a behaviour factor of q = 4 was utilised by 102 % according to the relevant design rules in fully plastic design.

From the PGA and the natural frequency of the structure an amplitude of the base displacement of 47,7 mm was derived. With an elastic limit storey drift of 26,5 mm this corresponded to an factor of 1,8, by which the driven base amplitude was lager than the elastic limit storey drift.

This approach would be very complicated for a wide range of structures to investigate, since all the structures would have to be designed to very narrow limits as far as utilisation under a given load configuration is concerned. For the Knoedel/Hrabowski 2014 study we chose a different approach. We started with a frame which is very similar to the Celtikci et. al (2010) frame with 80 metric tons of mass per storey, the members themselves have zero masses. We used tubular members since they seemed to be adaptable to the structural needs in a more simple way by just having 2 free parameters, the outer diameter and the wall thickness. We adjusted the members such that for a arbitrarily chosen base shear, which was divided to the storeys according to the EC8 rules (e.g. 1/2; 1/3; 1/6 from top to bottom), the relevant members were within 90 % to 100 % of their elastic limit load. This configuration is associated with a characteristic horizontal head displacement.

For simplicity we did not use pinned bars in the V-braced storeys. We considered the structures more comparable, if we just insert a V-brace in a frame without changing the frame.

For the harmonic base excitation (drive) we used this characteristic displacement as amplitude, multiplied by the assumed behaviour factor of EC8. In this way we simulated vice versa the usual design procedure, where you consider a given seismic situation and divide the seismic actions by the behaviour factor. The structure is then designed to withstand these reduced actions in a linear design.

2.2 Elements

We used a 2-D ANSYS model with plastic BEAM23. For simplification all sections were chosen to be tubular so we can follow the structural needs without being restricted by the stepped dimensions of a profile table.

The element size was chosen in a range of 0,2 m to 0,5 m, i.e. 6 to 30 elements per member, which proved to be sufficient for the present analysis.

2.3 Material

We used steel with a simplified bi-linear constitutive law with isotropic hardening.
Young's modulus with 2,1 GPa was used up to a yield limit of 235 MPa. Above that we used a straight line to the ultimate tensile stress 360 MPa at assumed 20 % strain. Thus we receive an 'engineering plastic modulus' with a slope of
(360 MPa – 235 MPa) / 0,20 = 625 MPa
This approach has been used before, see eq. 15 in Knoedel/Hrabowski (2012).

2.4 Verification

The dynamical properties of the numerical model were tested and verified by means of different runs:

- system under statical self weight, acting horizontal, to estimate the natural cantilever frequency by use of a Rayleigh-Morleigh procedure

- undamped oscillation along 10 or 20 periods following a jumped base displacement (Sprungfunktion) to verify the Eigenfrequency of the system

- damped oscillation (decay) following a jumped base displacement to verify the actual damping

- exciting the system by a harmonic drive to check the limit amplitudes of the damped system

2.5 Time-History Analysis

We performed a straight forward Euler-type time integration, using 15 to 20 load steps within a period. Between these major steps automatic time stepping was used, which resulted in up to 100 sub steps in between.

As described in an earlier paper we switched off the drive after 10 periods (Knoedel/Hrabowski 2012) which does not quite meet the demands of EC8 but does not influence the results of the structural behaviour under investigation.

2.6 Behaviour Factor

For evaluating the behaviour factor we used the maximum response displacement peak of the structure, since loading of the structure is corresponding to this peak. In case of offset however, this peak does not coincide with the dynamic amplitude, so the amplitude is given as half of the displacement range for information.

3 Investigated Structures

model no	no of storeys	structure	q	remark
1	1	frame	–	elastic
2	1	frame	–	plastic
3	1	frame	–	linear dynamic, step function (decay)
4	1	frame	–	nonlinear geometry, linear material
5	1	frame	–	linear, dynamic, harmonic drive
6	1	frame	–	linear geometry, plastic, harmonic drive

7	1	frame	–	sections changed
8	5	frame	14,1 (q=4)	plastic, 10 periods driven see different stiffness configuration in Model 23
9	5	V	6,4 (q=2) 6,3 (q=2)	plastic, 10 periods driven
10	2	frame	10,1 (q=4)	
10 reptd	2	frame	5,4 (q=2) 13,6 (q=4)	
11	2	V	7,6 (q=2) 11,2 (q=4)	
12	2	frame top V base	10,8 (q=2) 8,4 (q=4)	
13	2	V top frame base	6,7 (q=2) 8,4 (q=4)	
14	3	frame	5,9 (q=2) 14,3 (q=4)	
15	3	frame top frame middle V base	5,9 (q=2) 10,0 (q=4)	
16	3	frame top V middle V base	6,4 (q=2) 6,1 (q=4)	
17	3	V	7,5 (q=2) 11,0 (q=4)	

18	3	V top V middle frame base	6,7 (q=2) 9,5 (q=4)	
19	3	V top frame middle frame base	5,8 (q=2) 10,9 (q=4)	
20	3	V top frame middle V base	7,4 (q=2) 10,5 (q=4)	
21	3	frame top V middle frame base	6,4 (q=2) 8,6 (q=4)	
22	1	frame	6,2 (q=2) 7,9 (q=4)	
23	5	frame	5,9 (q=2) 8,3 (q=4)	
24	1	V	7,2 (q=2) 10,8 (q=4)	

4 Cross Sections and Properties

4.1 CHS properties

Stiffness of the tubes

$I = \pi * R^3 * T$

$R = (D - T)/2$

$A = 2\pi * R * T$

4.2 CHS 55x3

R = (5,5 cm – 0,3 cm)/2 = 2,6 cm

I = π * (2,6 cm)^3 * 0,3 cm = 16,6 cm^4

W = 16,6 cm^4 / 2,6 cm = 6,4 cm^3

A = 2π * 2,6 cm * 0,3 cm = 4,90 cm^2

N,R,k = 4,90 cm^2 * 235 N/mm^2 = 115 kN

M,gr,el = 6,4 cm^3 * 235 N/mm^2 = 1,50 kNm

4.3 CHS 60x2

R = (6,0 cm – 0,2 cm)/2 = 2,85 cm

I = π * (2,85 cm)^3 * 0,2 cm = 14,5 cm^4

W = 14,5 cm^4 / 2,85 cm = 5,1 cm^3

A = 2π * 2,85 cm * 0,2 cm = 3,58 cm^2

N,R,k = 3,58 cm^2 * 235 N/mm^2 = 84,2 kN

M,gr,el = 5,1 cm^3 * 235 N/mm^2 = 1,20 kNm

4.4 CHS 65x2

R = (6,5 cm – 0,2 cm)/2 = 3,15 cm

I = π * (3,15 cm)^3 * 0,2 cm = 19,6 cm^4

W = 19,6 cm^4 / 3,15 cm = 6,2 cm^3

A = 2π * 3,15 cm * 0,2 cm = 3,96 cm^2

N,R,k = 3,96 cm^2 * 235 N/mm^2 = 93,1 kN

M,gr,el = 6,2 cm^3 * 235 N/mm^2 = 1,46 kNm

4.5 CHS 80x3

R = (8 cm – 0,3 cm)/2 = 3,85 cm

I = π * (3,85 cm)^3 * 0,3 cm = 53,8 cm^4

W = 53,8 cm^4 / 3,85 cm = 14,0 cm^3

A = 2π * 3,85 cm * 0,3 cm = 7,26 cm^2

N,R,k = 7,26 cm^2 * 235 N/mm^2 = 170,5 kN

M,gr,el = 17,8 cm^3 * 235 N/mm^2 = 4,18 kNm

4.6 CHS 90x3

R = (9 cm – 0,3 cm)/2 = 4,35 cm

I = π * $(4,35\ cm)^3$ * 0,3 cm = 77,6 cm^4

W = 77,6 cm^4 / 4,35 cm = 17,8 cm^3

A = 2π * 4,35 cm * 0,3 cm = 8,20 cm^2

N,R,k = 8,2 cm^2 * 235 N/mm^2 = 192,7 kN

M,gr,el = 17,8 cm^3 * 235 N/mm^2 = 4,18 kNm

4.7 CHS 120x3

R = (12 cm – 0,3 cm)/2 = 5,85 cm

I = π * $(5,85\ cm)^3$ * 0,3 cm = 189 cm^4

W = 189 cm^4 / 5,85 cm = 32,3 cm^3

A = 2π * 5,85 cm * 0,3 cm = 11,0 cm^2

N,R,k = 11 cm^2 * 235 N/mm^2 = 259 kN

M,gr,el = 32,3 cm^3 * 235 N/mm^2 = 7,59 kNm

4.8 CHS 140x12

R = (14 cm – 1,2 cm)/2 = 6,4 cm

I = π * $(6,4\ cm)^3$ * 1,2 cm = 988 cm^4

W = 988 cm^4 / 6,4 cm = 154 cm^3

A = 2π * 6,4 cm * 1,2 cm = 48,3 cm^2

N,R,k = 48,3 cm^2 * 235 N/mm^2 = 1134 kN

M,gr,el = 154 cm^3 * 235 N/mm^2 = 36,3 kNm

4.9 CHS 150x12

R = (15 cm – 1,2 cm)/2 = 6,9 cm

I = π * (6,9 cm)^3 * 1,2 cm = 1238 cm^4

W = 1238 cm^4 / 6,9 cm = 179 cm^3

A = 2π * 6,9 cm * 1,2 cm = 52,0 cm^2

N,R,k = 52,0 cm^2 * 235 N/mm^2 = 1223 kN

M,gr,el = 179 cm^3 * 235 N/mm^2 = 42,2 kNm

4.10 CHS 160x12

R = (16 cm – 1,2 cm)/2 = 7,4 cm

I = π * (7,4 cm)^3 * 1,2 cm = 1528 cm^4

W = 1528 cm^4 / 7,4 cm = 206 cm^3

A = 2π * 7,4 cm * 1,2 cm = 55,8 cm^2

N,R,k = 55,8 cm^2 * 235 N/mm^2 = 1310 kN

M,gr,el = 206 cm^3 * 235 N/mm^2 = 48,5 kNm

4.11 CHS 240x12

R = (24 cm – 1,2 cm)/2 = 11,4 cm

I = π * (11,4 cm)^3 * 1,2 cm = 5585 cm^4

W = 5585 cm^4 / 11,4 cm = 490 cm^3

A = 2π * 11,4 cm * 1,2 cm = 85,6 cm^2

For information:

HEA 240:
I = 7760 cm^4
W = 645 cm^3

N,R,k = 85,6 cm^2 * 235 N/mm^2 = 2010 kN

M,gr,el = 490 cm^3 * 235 N/mm^2 = 115,2 kNm

4.12 CHS 260x12

R = (26 cm – 1,2 cm)/2 = 12,4 cm

I = π * (12,4 cm)^3 * 1,2 cm = 7190 cm^4

W = 7190 cm^4 / 12,4 cm = 580 cm^3

A = 2π * 12,4 cm * 1,2 cm = 93,5 cm^2

N,R,k = 93,5 cm^2 * 235 N/mm^2 = 2200 kN

M,gr,el = 580 cm^3 * 235 N/mm^2 = 136,3 kNm

4.13 CHS 280x12

R = (28 cm – 1,2 cm)/2 = 13,4 cm

I = π * (13,4 cm)^3 * 1,2 cm = 9070 cm^4

W = 9070 cm^4 / 13,4 cm = 677 cm^3

A = 2π * 13,4 cm * 1,2 cm = 101 cm^2

N,R,k = 101 cm^2 * 235 N/mm^2 = 2370 kN

M,gr,el = 677 cm^3 * 235 N/mm^2 = 159,1 kNm

4.14 CHS 300x12

R = (30 cm – 1,2 cm)/2 = 14,4 cm

I = π * (14,4 cm)^3 * 1,2 cm = 11260 cm^4

W = 11260 cm^4 / 14,4 cm = 782 cm^3

A = 2π * 14,4 cm * 1,2 cm = 109 cm^2

N,R,k = 109 cm^2 * 235 N/mm^2 = 2560 kN

M,gr,el = 782 cm^3 * 235 N/mm^2 = 183,8 kNm

4.15 CHS 400x12

CHS 400x12

R = (40 cm – 1,2 cm)/2 = 19,4 cm

I = π * (19,4 cm)^3 * 1,2 cm = 27500 cm^4

W = 27500 cm^4 / 19,4 cm = 1420 cm^3

A = 2π * 19,4 cm * 1,2 cm = 146,3 cm^2

N,R,k = 146 cm^2 * 235 N/mm^2 = 3430 kN

M,gr,el = 1420 cm^3 * 235 N/mm^2 = 333,7 kNm

4.16 CHS 450x12

CHS 450x12

R = (45 cm – 1,2 cm)/2 = 21,9 cm

I = π * $(21,9 \text{ cm})^3$ * 1,2 cm = 39600 cm^4

W = 39600 cm^4 / 21,9 cm = 1800 cm^3

A = 2π * 21,9 cm * 1,2 cm = 165 cm^2

N,R,k = 165 cm^2 * 235 N/mm^2 = 3880 kN

M,gr,el = 1800 cm^3 * 235 N/mm^2 = 423 kNm

4.17 CHS 1000x12

R = (100 cm – 1,2 cm)/2 = 49,4 cm

I = π * $(49,4 \text{ cm})^3$ * 1,2 cm = 454500 cm^4

(80 times bigger than smaller tube)

W = 454500 cm^4 / 49,4 cm = 9200 cm^3

(19 times bigger than 240x12)

A = 2π * 49,4 cm * 1,2 cm = 372 cm^2

N,R,k = 372 cm^2 * 235 N/mm^2 = 8742 kN

M,gr,el = 9200 cm^3 * 235 N/mm^2 = 2162 kNm

4.18 Celtikci HEA 320

I = 22930 cm^4

W = 1480 cm^3

A = 124 cm^2

4.19 Celtikci IPE 450

$I = 33740 \text{ cm}^4$

$W = 1500 \text{ cm}^3$

$A = 99 \text{ cm}^2$

5 Models

5.1 Model 1 – elastic, static

5.1.1 General

single storey

5 m	frame width
3 m	frame height

element divisor 1 m / 5

steel elastic

no density

0,0167	damping constant
1000x12	tubular waler
240x12	tubular columns
36.000 kgs	mass added in each of the frame's corners
dx,dy	boundary conditions: feet displacements blocked
9,81 m/s^2	gravity loads

5.1.2 Horizontal stiffness

1.e+06	horizontal load in left corner
382,8 mm	associated horizontal displacement
382,8 mm	same with EFAK 50

calculatory horizontal displacement with infinitely stiff waler

$c = 2 * 3 * EI / L^3$

$c = 2 * 3 * 2{,}1*10^5 \text{ N/mm}^2 * 5585 \text{ cm}^4 / (3 \text{ m})^3 = 2606 \text{ N/mm} = 2{,}61*10^6 \text{ N/m}$

$u = F / c$

$u = 1.e{+}06 \text{ N} / 2{,}61*10^6 \text{ N/m} = 383 \text{ mm}$

5.1.3 Natural frequency

$\omega = \sqrt{(c/m)}$

$\omega = \sqrt{(2{,}61*10^6 \text{ N/m} / 72000 \text{ kgs})} = 6{,}02 \text{ rad/s}$

$f = \omega / 2\pi$

$f = 6{,}02 \text{ rad/s} / 2\pi = 0{,}958 \text{ /s}$

$T = 1 / 0{,}958 \text{ /s} = 1{,}044 \text{ s}$

Detuning due to damping

$k = \sqrt{(1 - D^2)}$

$k = \sqrt{(1 - 0{,}05^2)} = 0{,}9987$ or 1,3 %o relative difference

Driving period:

T,drive = 1,044 s / 0,999 = 1,043

5.1.4 Stress design

Horizontal force in/at waler

F = 100 kN

Base shear per column

F = 50 kN

Corner moment

M = 50 kN * 3 m = 150 kNm

outer fibre stress

$\sigma = 150 \text{ kNm} / 490 \text{ cm}^3 = 306 \text{ N/mm}^2$

(in waler: $150 \text{ kNm} / 9200 \text{ cm}^3 = 16{,}3 \text{ N/mm}^2$)

Normal forces in columns from tilting

$F = 100 \text{ kN} * 3 \text{ m} / 5 \text{ m} = \pm 60 \text{ kN}$

Normal force in waler, depending on position of horizontal force: ±50 kN

All checked in detail OK.

5.2 Model 2 – plastic, static

Like Model 1 but additional:

Plastic constitutive law for S235

Horizontal force reduced to

$100\ kN * 235\ N/mm^2 / 306\ N/mm^2 * 1{,}2 = 92\ kN$

Stress:

$306\ N/mm^2 * 92\ kN / 100\ kN = 282\ N/mm^2$

Total strain

$282\ N/mm^2 / 2{,}1*10^5\ N/mm^2 = 1{,}34*10^{-3}$

Elastic limit strain

$235\ N/mm^2 / 2{,}1*10^5\ N/mm^2 = 1{,}12*10^{-3}$

Plastic strain

$1{,}34*10^{-3} - 1{,}12*10^{-3} = 2{,}21*10^{-4} = 0{,}221*10^{-3}$

ANSYS result (EFAK = 5): $0{,}681*10^{-3}$

ANSYS result (EFAK = 50): $0{,}682*10^{-3}$

Storey drift

382,8 mm * 92 kN / 1000 kN = 35,2 mm

ANSYS result (EFAK = 5): 38,4 mm

ANSYS result (EFAK = 50): 38,3 mm

5.3 Model 3 – linear, dynamic, step function

Like Model 1 but

geometry linear, material linear

1000 m foot displacement as step function

20 increments per period of 1,043 seconds length

Damper 0,0167, should correspond to Leer's damping constant of 0,05 = 5 %

First transition of the 1000 m line

time [s]	displacement [m]
0,261	879
0,283 (interpolated)	1000
0,313	1160

Last transition of the 1000 m line after 13,5 periods

time [s]	displacement [m]
14,602	1000,4
14,607 (interpolated)	1000
14,654	996,4

Length of period

T = (14,607 s – 0,283 s) / 13,5 = 1,061

Results with EFAK=50 after 14 seconds identical!

5.4 Model 4 – geometrically nonlinear, dynamic, step function

Like Model 3 but

nonlinear geometry, material linear

100 mm foot displacement as step function

20 increments per period of 1,043 seconds length

NSUB,10,100,2,1

NEQIT,50

Damper 0,0167, should correspond to Leer’s damping constant of 0,05 = 5 %

First transition of the 100 mm line

time [s]	displacement [mm]
0,261	88,55
0,282 (interpolated)	100
0,313	117,0

Last transition of the 100 mm line after 13,5 periods

time [s]	displacement [mm]
14,498	100,14
14,516 (interpolated)	100
14,550	99,73

Length of period

T = (14,516 s – 0,282 s) / 13,5 = 1,054

Results with EFAK=50 after 14 seconds:

time [s]	displacement [mm]
14,498	100,07
14,507 (interpolated)	100

14,550	99,67

Length of period

T = (14,507 s – 0,282 s) / 13,5 = 1,0537

can be taken as identical

Estimate of Lehr's damping number, input 0,0167

Formula of Mensinger: $\delta = \ln (Amp,i / Amp,i+1)$

time [s]	displacement [mm]	log. dec. δ	Lehr's number D
0,522	185	0,139	0,022
1,56	161	0,105	0,017
2,66	145	0,105	0,016
3,70	133	0,086	0,014
4,80	123	0,078	0,012
5,79	118	0,041	0,007
6,88	113	0,043	0,007
7,93	109	0,036	0,006
8,97	107	0,019	0,003
10,01	105,0	0,019	0,003
11,06	103,6	0,013	0,002
12,15	102,6	0,010	0,002
13,19	101,9	0,007	0,001

14,24	101,4	0,005	0,001

Lehr's number is

$D = \delta / 2\pi$

5.5 Sub-Summary

Geometrical linear analysis with EFAK=5 are sufficient to get satisfactory results

The above results for Ninc = 20 show end of 13th period at 13,98 seconds

T = (13,98 s – 0,282 s) / 13,0 = 1,054 q.e.d.

A test with Ninc = 10 shows end of 13,0th period at 14,44 seconds.

T = (14,44 s – 0,282 s) / 13,0 = 1,089

A test with Ninc = 15 shows end of 13,0th period at 14,16 seconds.

T = (14,16 s – 0,282 s) / 13,0 = 1,068

A test with Ninc = 18 shows end of 13,0th period at 14,10 seconds.

T = (14,10 s – 0,282 s) / 13,0 = 1,063

Ninc = 15 is taken to be sufficient.

A period of 1,068 will be taken as drive.

5.6 Model 5 – linear, dynamic, harmonic drive

Like Model 4 but

Harmonic driven foot displacements with 47,7 mm amplitude and 1,068 s driving period.

Damping input 0,0167.

Maximum structural answer 467 mm after 19,7 seconds.

Re-calculated damping number is derived from amplification factor:

k,amp = A,response / A,drive = 467 mm / 47,7 mm = 9,79

D = 1 / 2*k,amp

D = 1 / 2 * 9,79 = 0,051

The reasons for the difference of this steady state (effective) damping and the decreasing damping number in the above test is not investigated in this paper. The input number 0,0167 for the damping seems to be sufficient to produce the desired results.

Interesting though, that the steady state amplitudes +444 mm / –446 mm after 10 seconds are by some 5 % smaller than found by Knoedel/Hrabowski (2012), this is maybe due to an non optimum drive (still more elements and smaller time stepping). However for the scope of this paper the behaviour seems to be satisfactory.

5.7 Model 6 – geometric linear, plastic, dynamic, harmonic drive

Like Model 5 but

Plastic constitutive law as known

/CONFIG,NRES,15000
NEQIT,25
NSUB,30,1000,30,1

I-O-time 18:54 to 19:06 for 12 seconds with NINC=15.

5.8 Model 7 – columns and waler interchanged

Like Model 6 but

columns with “infinite stiffness” (i.e. tube 1000x12) and waler with tube 240x12.
Should give same results when increasing span to 6 m.

... proved

5.9 Model 8 – Celtikci-frame

5.9.1 Properties

6 m width of frame

4 m height of storey

5 number of storeys

400x12-S235-tubular members corresponding to HEA 320 columns

the section modulus holds for IPE 450 walers as well

8000 kgs mass at each column-waler-joint, gives 16 metric tons per storey and a total mass of 80 metric tons on the frame

157 kN horizontal loads from self weight per storey (Rayleigh-Morleigh loads)

627 mm horizontal deflection of top midspan node nm5 under RM-loads

392,5 kN base shear per column

1570 kNm bending moment in the first storey top end column confirmed by ANSYS

433 kNm ANSYS result: moment in second storey bottom end column

2003 kNm ANSYS result for first storey waler end moment

333,7 kNm elastic limit moment (see section Cross Sections)

elastic limit top storey drift is determined by waler:

627 mm * 333,7 kNm / 2003 kNm = 104,5 mm

5.9.2 Dynamics

$\omega = \sqrt{(g / w_{,max})}$

$\omega = \sqrt{(9{,}81\ m/s^2 / 0{,}627\ m)} = 3{,}96\ rad/s$

$f = \omega / 2\pi$

$f = 3{,}96\ rad/s / 2\pi = 0{,}6295\ Hz$

$T = 1{,}589\ s$

Step function base displacement 100 mm, 20 sec time history analysis.

Counting the last 5 of 13 periods.

damping 0,0167

Transition of the 100 mm line after 12,0764

time [s]	displacement [mm]
12,0764	95,04
12,1398 (interpolated)	100
12,1559	101,26

Last transition of the 100 mm line after another 5 periods

time [s]	displacement [mm]
19,4653	99,67
19,4787 (interpolated)	100
19,5447	101,63

Length of period

T = (19,4787 s – 12,1398 s) / 5 = 1,4678 s

Relation to Rayleigh-Morleigh evaluation

(1,4678 s – 1,589 s) / 1,589 s = –0,0763

ANSYS natural frequency under damping is by 7,63 % smaller than Rayleigh-Morleigh estimation.

Same with zero damping

time [s]	displacement [mm]
12,0764	81,68
12,1125 (interpolated)	100
12,1559	122,08

Last transition of the 100 mm line after another 5 periods

time [s]	displacement [mm]
19,3858	55,99
19,4580 (interpolated)	100
19,4653	104,48

Length of period

T = (19,4580 s – 12,1125 s) / 5 = 1,4691 s

Relation to Rayleigh-Morleigh evaluation

(1,4691 s – 1,589 s) / 1,589 s = –0,0755

ANSYS natural frequency without damping is by 7,55 % smaller than Rayleigh-Morleigh estimation.

Relation of the ANSYS periods with and without damping:

(1,4678 s – 1,4691 s) / 1,4691 s = 0,0008849 app. 0,9 %o, “should” be 1,3 %o.

Undamped excitation with 0,418 m:

after 19 periods some 29,6 m amplitude

(looks like linear progression of the amplitudes from peak to peak).

Damped (DAMP 0,0167; should correspond to 5 %) excitation with 0,418 m:

after 19 periods some 6,76 m amplitude

Amplification should be

1 / 2D = 1 / (2 * 0,05) = 10

Actual amplification is

6,76 m / 0,418 m = 16,2

which means, that the damping is a bit smaller than assumed. This is

a) on the safe side, and

b) does not matter as it comes to plastic response, see earlier papers.

5.9.3 Seismic Design

With an elastic limit top storey drift of 104,5 mm

and an assumed behaviour factor of 4

the base displacement amplitude should be 418 mm.

Run with plastic options showed a considerable offset.

Within a 5 seconds run (12 min I/O) the maximum amplitudes were

+479 mm / –126 mm

mean: 177 mm; amplitude 303 mm

Actual amplification is

479 mm / 418 mm = 1,15

or 303 mm / 418 mm = 0,725 ?

Behaviour factor q is

q = 6760 mm / 479 mm = 14

20 seconds run (45 Min I/O)

+476 mm at 0,88 sec, load step 10 substep 101

+483 mm at 2,35 sec, load step 25 substep 101 (!)

+467 mm at 14,1 sec, load step 145 substep 104

–255 mm at 14,8 sec, load step 152 substep 104 (!)

mean of the last two major peaks: 106 mm; amplitude 361 mm

Actual amplification is

483 mm / 418 mm = 1,16

or 361 mm / 418 mm = 0,864 ?

Behaviour factor q is

q = 6760 mm / 483 mm = 14,1

5.9.4 Convergence

The moment convergence tolerance was set to

CNVTOL, M,,0.01,2

in order to get the thing going.

(A time around 2,37 seconds seemed to be critical; several attempts to get beyond this point.)

Trying to remove damping, which should be insignificant with plastic calculations (see earlier papers) produced non-convergence in very early load steps.

Same, but trying to calculate more than 5 seconds, and trying to sharpen the M-TOL to

CNVTOL, M,,0.005,2

gave a non-convergence at 4,70 seconds.

Generally, non-convergence showed up as DOF oscillation with in a substep,

e.g. +1.E-4 and –1.E-4.

5.10 Model 9 – V-Bracing

5.10.1 Properties

Similar as Model 8 but V-bracing

6 m width of frame

4 m height of storey

5 number of storeys

400x12-S235 tubular members for columns

400x12-S235 tubular members for walers

120x3-S235 tubular members for diagonals

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8000 kgs mass at each column-waler-joint, gives 16 metric tons per storey and a total mass of 80 metric tons on the frame

157 kN horizontal loads from self weight per storey (Rayleigh-Morleigh loads)

75,6 mm horizontal deflection of top midspan node nm5 under RM-loads

589 N/mm^2 in the bottom tension diagonal

551 N/mm^2 in the bottom compression diagonal

elastic limit top storey drift is determined by bottom diagonal:
75,6 mm * 235 N/mm^2 / 589 N/mm^2 = 30,2 mm

5.10.2 Dynamics

$\omega = \sqrt{(g / w,max)}$
$\omega = \sqrt{(9{,}81\ m/s^2 / 0{,}0756\ m)} = 11{,}4$ rad/s

$f = \omega / 2\pi$
$f = 11{,}4$ rad/s $/ 2\pi = 1{,}813$ Hz
$T = 0{,}5516$ s

Step function base displacement 100 mm, 10 sec time history analysis with EFAK 5 and Ninc 50.
Counting the last 10 of 19 periods.
damping 0

Transition of the 100 mm line after 4,61 sec

time [s]	displacement [mm]
4,613	59,98
4,6297 (interpolated)	100
4,642	129,3

Transition of the 100 mm line after 9,6 sec

time [s]	displacement [mm]
9,608	54,58
9,630 (interpolated)	100
9,637	113,8

Length of period

T = (9,630 s – 4,6297 s) / 10 = 0,5000 s

Relation to Rayleigh-Morleigh evaluation

(0,5000 s – 0,5516 s) / 0,5516 s = –0,0935

ANSYS natural frequency without damping is by 9,35 % smaller than Rayleigh-Morleigh estimation. This may be due to the fact, that the assumption of Rayleigh-Morleigh – the elastica of the static loading should be identical to the elastica in the reversal point – might be violated.

By trial and error a damping number of 0,001 was established where the damping behaviour is similar to 5-%-damping.

Undamped excitation with 0,0604 m:

after 39 periods and 19,7 sec some 8,23 m amplitude

(looks like linear progression of the amplitudes from peak to peak).

Amplification should be

1 / 2D = 1 / (2 * 0,05) = 10

Amplitudes should be app. 0,60 m for an amplification by a factor of 10.

Damped (DAMP 0,001; should correspond to app. 5 %) excitation with 0,0604 m:

after 39 periods and 19,73 sec some 4,35 m amplitude.

Damped (DAMP 0,005; should correspond to app. 5 %) excitation with 0,0604 m:

after 39 periods and 19,73 sec some 1,18 m amplitude.

Damped (DAMP 0,010; should correspond to app. 5 %) excitation with 0,0604 m:
after 39 periods and 19,75 sec some 0,601 m amplitude.

Actual amplification is
601 mm / 60,4 mm = 9,95

5.10.3 Seismic Design

With an elastic limit top storey drift of 30,2 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 60,4 mm.

Run with plastic options showed a considerable offset.
Within a 1,5 seconds run (30 min I/O) the maximum amplitude was
+94,3 mm
(funny that this is smaller than the next one, because this was generated with DAMP 0,001, the next one with DAMP 0,01)

Actual amplification is
94,3 mm / 60,4 mm = 1,56

Behaviour factor q is
q = 601 mm / 94,3 mm = 6,4

Within a 7 seconds run (120 min I/O) but only driven for 5 periods, the first to response peaks were the major ones
+95,7 mm / –63,6 mm

Actual amplification is
95,7 mm / 60,4 mm = 1,58

Behaviour factor q is
q = 601 mm / 95,7 mm = 6,3

Another run over 7 seconds with 10 driven periods (120 min I/O) shows same results: after the first two major peaks as described above only smaller peaks follow.

+95,7 mm at 0,2500 sec, load step 11 substep 101

–63,6 mm at 0,5250 sec, load step 22 substep 100

Last peaks before the response decays are

–35,3 mm at 5,075 sec, load step 204, substep 100

+41,1 mm at 5,525 sec, load step 222 substep 100

Actual amplification is

95,7 mm / 60,4 mm = 1,58

Behaviour factor q is

q = 601 mm / 95,7 mm = 6,3

5.11 Model 10 – see amendment

5.11.1 Properties

Like Model 8 but only 2 storeys

Seismic design with respect to different storey loads:

1/3 in storey 1

2/3 in storey 2

Base shear 102 kN produces max stress 233 MPa in the first waler.

Head displacement (storey drift) is 69,7 mm.

Max stress in the second waler is 107 MPa, i.e. utilisation 0,455.

Second waler should have a section modulus of

0,455 * 1420 cm3 = 646 cm3

chosen: CHS 280x12 having W = 677 which is an overmatch of 1,05

… this seems to be statically highly indetermined, so a trial and error procedure was used to find:

1. storey columns	400x12
1. storey waler	400x12
2. storey columns	260x12
2. storey waler	160x12

a configuration where the walers had differences in the stresses of 2,8 %.

At a total of 80 kN base shear the max stresses were at 233 MPa and head displacement is 97,3 mm.

5.11.2 Dynamics

Natural frequency of the system is (damping 0,005)

Estimated: 0,25 secs to 10,7 secs with 16 periods.

T = 0,653

A driven run shows unexpected non-linear features.

Ampli=0,1; Periode=0,653; Dauer=20; Ninc=100;

max response amplitude: 307 mm at second positive peak

A decay analysis with DAMP 0,005 showed a decrease of more than half the amplitude after 10 seconds or app. 15 periods. This corresponds to an effective damping of 0,053.

Natural frequency of the system is (damping 0,001):

time [s]	displacement [mm]
0,2612	97,3
0,2630 (interpolated)	100
0,3047	162,3

time [s]	displacement [mm]
9,229	86,7
9,251 (interpolated)	100
9,273	112,8

Length of period

T = (9,251 s – 0,263 s) / 10 = 0,8988 s

Remark:

An explanation of the previously determined period of 0,653 sec could not be found. This error cost 24 hrs delay.

A driven run with base displacement 0,10 m and DAMP 0,001 over 20 secs goes up to 6 m amplitude and still is not in steady state. Estimated limit amplitude is 9 m, which would be an amplification of

V = 9,0 m / 0,10 m = 90

or delta = 0,03 or D = 0,0056

A driven run with base displacement 0,10 m and DAMP 0,005 over 20 secs or 21 periods goes up to 2,82 m amplitude and is not quite in steady state.

V = 2,82 m / 0,10 m = 28

or delta = 0,111 or D = 0,0177

A driven run with base displacement 0,10 m and DAMP 0,015 over 20 secs or 21 periods goes up to +1093 m / -1040 mm amplitude in seemingly steady state.

V = 1,07 m / 0,10 m = 10,7

or delta = 0,294 or D = 0,0467

5.11.3 Seismic Design

With an elastic limit top storey drift of 97,3 mm

and an assumed behaviour factor of 4

the base displacement amplitude should be 389 mm.

A driven run linear with DAMP 0,015 over 20 secs or 21 periods goes up to +4253 m / -4047 mm amplitude in seemingly steady state. Steady state is reached after about 10 periods.

V = 4150 mm / 389 mm = 10,7

or delta = 0,294 or D = 0,0467

A run over 12 seconds with 10 driven periods (13 min I/O) shows one major peak and then offsetting, so that the displacement range is growing bigger. Towards the end there is a higher mode such, that the first storey has a larger positive storey drift than the second storey.

Head displacements are recorded:

+295 mm at 0,49434 sec, load step 12 substep 11

–151 mm at 0,98868 sec, load step 23 substep 11

Last peaks before the response decays are

+142 mm at 8,58354 sec, load step 192, substep 12

–412 mm at 9,03294 sec, load step 202 substep 17

Displacement range at the beginning:

295 mm –(–151 mm) = 446 mm

corresponding amplitude is 223 mm

Displacement range at the end:

142 mm –(–412 mm) = 554 mm

corresponding amplitude is 277 mm

Actual amplification is

412 mm / 389 mm = 1,06

Behaviour factor q is

q = 4150 mm / 412 mm = 10,1

5.12 Model 11

5.12.1 Properties

Like Model 10 but with V-braces / or: like model 9 but only with 2 storeys

Seismic design with respect to different storey loads:

1/3 in storey 1

2/3 in storey 2

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	120x3
2. storey columns	90x3

2. storey waler 90x3

2. storey diagonal 90x3

Forces in the diagonals are verified with RSTAB for a base shear of 258 kN

143 kN upper

197 kN lower

Stresses should be

143 kN / 8,20 cm^2 = 174 N/mm^2

197 kN / 11,0 cm^2 = 179 N/mm^2

Verified in ANSYS under consideration of max and min stresses within the same bar.

Base shear 324 kN produces max stress 235 MPa in the first storey tension diagonals, the second storey are around 6 % smaller.

Head displacement (storey drift) is 18,0 mm.

5.12.2 Dynamics

Rayleigh-Morleigh estimation of the natural frequency:

15,6 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 3,99$

$T = 0,2506$

Natural frequency of the system is (damping 0,001)

time [s]	displacement [mm]
0,0625	83,0
0,068 (interpolated)	100
0,075	120,5

time [s]	displacement [mm]
4,888	96,8

4,896 (interpolated)	100
4,900	101,4

Length of period

T = (4,896 s – 0,068 s) / 20 = 0,2414 s.

A driven run with base displacement 0,10 m and DAMP 0,001 over 10 secs goes up to

+3785 mm / –3790 mm

and still almost in steady state.

V = 3,8 m / 0,10 m = 38

or delta = 0,083 or D = 0,0132

A driven run with base displacement 0,10 m and DAMP 0,004 over 10 secs goes up to

+1021 mm / –1021 mm

V = 1,02 m / 0,10 m = 10,2

or delta = 0,308 or D = 0,0049

5.12.3 Seismic Design

With an elastic limit top storey drift of 18,0 mm

and an assumed behaviour factor of 2

the base displacement amplitude should be 36 mm.

A driven run with base displacement 36 mm and DAMP 0,004 over 10 secs goes up to

+368 mm / –368 mm

V = 368 mm / 36 mm = 10,2

or delta = 0,308 or D = 0,049

A plastic run over 3 seconds with 10 driven periods (9 min I/O) shows one major positive peak and then offsetting, so that the displacement range is growing bigger. All the time the displacements of the first storey are more symmetric, so that the negative peaks of the first storey are bigger than those of the second storey.

Head displacements are recorded:

+48,5 mm at 0,14484 sec, load step 13 substep 11

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–20,7 mm at 0,20519 sec, load step 18 substep 15 (negative peak of first storey)

Last peaks before the response decays are

+44,4 mm at 2,31744 sec, load step 193, substep 11

–15,4 mm at 2,43814 sec, load step 203 substep 11

Displacement range at the beginning:

app. 50 mm

corresponding amplitude is 25 mm

Displacement range at the end:

44,4 mm –(–15,4 mm) = 59,8 mm

corresponding amplitude is 29,9 mm

Actual amplification is

48,5 mm / 36 mm = 1,35

Behaviour factor q is

q = 368 mm / 48,5 mm = 7,6

Amended on the 09.03.2014

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*36 mm = 72 mm

A driven run with base displacement 72 mm and DAMP 0,004 over 10 secs goes up to

+735,2 mm / –735,2 mm

V = 735,2 mm / 72 mm = 10,2

or delta = 0,308 or D = 0,049

A plastic run over 3 seconds with 10 driven periods (7 min I/O) shows one major positive peak and then offsetting, so that the displacement range is growing bigger. All the time the displacements of the first storey are more symmetric, so that the negative peaks of the first storey are bigger than those of the second storey.

Head displacements are recorded:

+65,6 mm at 0,15691 sec, load step 14 substep 12

Last peaks before the response decays are

+57,6 mm at 2,31744 sec, load step 193, substep 11

–11,9 mm at 2,43814 sec, load step 203 substep 10

Displacement range at the end:

57,6 mm –(–11,9 mm) = 69,5 mm

corresponding amplitude is 34,8 mm

Actual amplification is

65,6 mm / 72 mm = 0,91

Behaviour factor q is

q = 735,2 mm / 65,6 mm = 11,2

5.13 Model 12

5.13.1 Properties

Mix up of Model 10 and Model 11: two storey frame with V-brace in the basement.

Seismic design with respect to different storey loads:

1/3 in storey 1

2/3 in storey 2

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	55x3
2. storey columns	260x12
2. storey waler	160x12
2. storey diagonal	–

Base shear 135 kN produces max stress 236 MPa in the first storey tension diagonals, the second storey bending stresses at the feet of the columns are the same, bending stresses in the top edges are around 3 % smaller.

Head displacement (storey drift) is 58,2 mm.

5.13.2 Dynamics

Rayleigh-Morleigh estimation of the natural frequency:

109 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 1{,}51$

$T = 0{,}66$

Natural frequency of the system is (damping 0,001, 20 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,135	45,0
0,180 (interpolated)	100
0,180	100,3

time [s]	displacement [mm]
19,215	89,1
19,247 (interpolated)	100
19,260	104,5

Length of period

$T = (19{,}247 \text{ s} - 0{,}180 \text{ s}) / 30 = 0{,}6356 \text{ s}.$

A driven run with base displacement 0,10 m and DAMP 0,001 over 20 secs goes up to

+5818 mm / –5789 mm

and is not in steady state.

$V = 5{,}8 \text{ m} / 0{,}10 \text{ m} = 58$

or delta = 0,054 or D = 0,0086

A driven run with base displacement amplitude 0,10 m and DAMP 0,004 over 20 secs goes up to +2475 mm / –2474 mm

V = 2,47 m / 0,10 m = 24,7

or delta = 0,127 or D = 0,0202

5.13.3 Seismic Design

With an elastic limit top storey drift of 58,2 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 116 mm.

A driven run with base displacement 116 mm and DAMP 0,010 over 20 secs goes up to +1258 mm / –1258 mm

V = 1258 mm / 116 mm = 10,8

or delta = 0,290 or D = 0,046

A run over 10 seconds with 10 driven periods (14 min I/O) shows head displacements with offset but the amplitudes are increasing towards the end.

Head displacements of last peaks before decay:
+153,2 mm at 6,06998 sec, load step 192, substep 19
–84,4 mm at 6,38778 sec, load step 202 substep 13

Displacement range at the end:
153,2 mm –(–84,4 mm) = 237,6 mm
corresponding amplitude is 119 mm

Actual amplification is
153,2 mm / 116 mm = 1,32

Behaviour factor q is
q = 1258 mm / 116 mm = 10,8

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*116 mm = 232 mm

A driven run with base displacement 232 mm and DAMP 0,010 over 10 secs goes up to

+2516 mm / –2516 mm

V = 2516 mm / 232 mm = 10,8

or delta = 0,290 or D = 0,046

A run over 10 seconds with 10 driven periods (14 min I/O) shows head displacements with offset but the amplitudes are increasing towards the end.

Head displacements of last peaks before decay:

+301,3 mm at 6,06998 sec, load step 192, substep 15

+6,9 mm at 6,38778 sec, load step 202 substep 16

Displacement range at the end:

301,3 mm –(+6,9 mm) = 294,4 mm

corresponding amplitude is 147 mm

Actual amplification is

301,3 mm / 232 mm = 1,30

Behaviour factor q is

q = 2516 mm / 301 mm = 8,4

5.14 Model 13

5.14.1 Properties

Like Model 12, two storey frame – but V-brace in the second storey.

Seismic design with respect to different storey loads:

1/3 in storey 1

2/3 in storey 2

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	–
2. storey columns	65x2

2. storey waler 65x2

2. storey diagonal 65x2

Base shear 155 kN produces max stress 234 MPa in the first storey column top end and in the second storey tension diagonal. The second storey compression diagonal is at –217 MPa “max stress” and –234 MPa “min stress”.

Head displacement (storey drift) is 59,3 mm.

5.14.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

184 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 1{,}16$

$T = 0{,}86$

Natural frequency of the system is (damping 0,001, 20 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,160	72,4
0,187 (interpolated)	100
0,200	113,8

time [s]	displacement [mm]
19,760	92,8
19,781 (interpolated)	100
19,800	106,6

Length of period

$T = (19{,}781 \text{ s} - 0{,}187 \text{ s}) / 29 = 0{,}6757 \text{ s}.$

A driven run with base displacement amplitude 0,119 m and DAMP 0,010 over 20 secs goes up to +1292 mm / –1292 mm

V = 1292 mm / 119 mm = 10,9

or delta = 0,289 or D = 0,046

5.14.3 Seismic Design

With an elastic limit top storey drift of 59,3 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 119 mm.

A plastic run over 10 seconds with 10 driven periods (11 min I/O) shows head displacements with offset. First positive is a local high but the amplitudes are increasing towards the end.
Head displacements of first positive and last peaks before decay:
+181,1 mm at 0,371635 sec, load step 12, substep 11
+192,1 mm at 6,452935 sec, load step 192, substep 19
–55,7 mm at 6,790785 sec, load step 202 substep 23

Displacement range at the end:
192,1 mm –(–55,7 mm) = 247,8 mm
corresponding amplitude is 124 mm

Actual amplification is
192,1 mm / 119 mm = 1,61

Behaviour factor q is
q = 1292 mm / 192 mm = 6,7

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*119 mm = 238 mm

A driven run with base displacement 238 mm and DAMP 0,010 over 20 secs goes up to +2583 mm / –2583 mm

V = 2583 mm / 238 mm = 10,9

or delta = 0,289 or D = 0,046

A plastic run over 10 seconds with 10 driven periods (12 min I/O) shows offset and increasing amplitudes towards the end.

Head displacements of last peaks before decay:

+309,4 mm at 6,452935 sec, load step 192, substep 12

+3,1 mm at 6,790785 sec, load step 202 substep 16

Displacement range at the end:

309,4 mm –(+3,1 mm) = 306,3 mm

corresponding amplitude is 153 mm

Actual amplification is

309,4 mm / 238 mm = 1,30

Behaviour factor q is

q = 2583 mm / 309 mm = 8,4

5.15 Model 14

5.15.1 Properties

Like Model 8 but three storeys.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
2. storey columns	260x12
2. storey waler	260x12
3. storey columns	170x12
3. storey waler	170x12

Base shear 100 kN produces max stress

209 MPa in the first storey waler

217 MPa in the second storey waler

230 MPa in the third storey waler

Based on the max stresses in the third storey waler this is a range of 9 %.

Based on the above results the elastic limit base shear is found to be 102 kN at a head displacement of 159 mm.

5.15.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

594 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 0,647$

$T = 1,546$

Natural frequency of the system is (damping 0,001, 30 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,3716	90,7
0,3814 (interpolated)	100
0,4054	122,9

time [s]	displacement [mm]
28,886	86,3
28,908 (interpolated)	100
28,920	107,6

Length of period

$T = (28,908 \text{ s} - 0,381 \text{ s}) / 21 = 1,3584 \text{ s}.$

A driven run with base displacement amplitude 0,318 m and DAMP 0,010 over 40 secs goes up to +8390 mm / –8396 mm

V = 8390 mm / 318 mm = 26,4

or delta = 0,119 or D = 0,019

A driven run with base displacement amplitude 0,318 m and DAMP 0,025 over 40 secs goes up to +3574 mm / –3574 mm

V = 3574 mm / 318 mm = 11,2

or delta = 0,280 or D = 0,044

A driven run with base displacement amplitude 0,318 m and DAMP 0,028 over 20 secs goes up to +3198 mm / –3197 mm

V = 3198 mm / 318 mm = 10,1

or delta = 0,312 or D = 0,050

5.15.3 Seismic Design

With an elastic limit top storey drift of 159 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 318 mm.

A plastic run over 20 seconds with 10 driven periods (12 min I/O) shows head displacements with offset with increasing head amplitudes.
Head displacements last peaks before decay:
+538,4 mm at 13,04064 sec, load step 193, substep 22
–201,8 mm at 13,65192 sec, load step 202 substep 20

Displacement range at the end:
538,4 mm –(–201,8 mm) = 720,2 mm
corresponding amplitude is 370 mm

Actual amplification is
538,4 mm / 318 mm = 1,69

Behaviour factor q is
q = 3198 mm / 538,4 mm = 5,9

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*318 mm = 636 mm

A driven run with base displacement amplitude 0,636 m and DAMP 0,028 over 30 secs goes up to +6408 mm / –6048 mm

V = 6408 mm / 636 mm = 10,1

or delta = 0,312 or D = 0,050

A plastic plastic run over 20 seconds with 10 driven periods (15 min I/O) shows one major positive head peak, then offset with increasing head amplitudes.

First positive head peak and last peaks before decay:

+447,0 mm at 0,88296 sec, load step 14, substep 16

+402,3 mm at 13,04064 sec, load step 193, substep 16

–409,9 mm at 13,71948 sec, load step 203 substep 12

Displacement range at the end:

402,3 mm –(–409,9 mm) = 812,2 mm

corresponding amplitude is 406 mm

Actual amplification is

447 mm / 636 mm = 0,703

Behaviour factor q is

q = 6408 mm / 447 mm = 14,3

5.16 Model 15

5.16.1 Properties

Like Model 14 with three storeys but a V-brace in the first storey.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	65x2
2. storey columns	260x12
2. storey waler	260x12
3. storey columns	170x12
3. storey waler	170x12

Base shear 100 kN produces max stress

211 MPa in the first storey diagonal

204 MPa in the second storey waler

232 MPa in the third storey column

Based on the max stresses in the third storey waler this is a range of 12 %.

Head displacement of 116 mm.

5.16.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

408 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 0{,}780$

$T = 1{,}28$

Natural frequency of the system is (damping 0,001, 30 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,300	70,9
0,329 (interpolated)	100
0,360	131,6

time [s]	displacement [mm]
29,340	84,1

29,382 (interpolated)	100
29,400	106,9

Length of period

T = (29,382 s – 0,329 s) / 25 = 1,1621 s.

A driven run with base displacement amplitude 0,232 m and DAMP 0,025 over 30 secs goes up to +2093 mm / –2093 mm

V = 2093 mm / 232 mm = 9,0

or delta = 0,349 or D = 0,056

A driven run with base displacement amplitude 0,232 m and DAMP 0,023 over 30 secs goes up to +2273 mm / –2273 mm

V = 2273 mm / 232 mm = 9,8

or delta = 0,321 or D = 0,051

5.16.3 Seismic Design

With an elastic limit top storey drift of 116 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 232 mm.

A plastic run over 15 seconds with 10 driven periods (14 min I/O) shows offset with steadily increasing head amplitudes.

Head displacements last peaks before decay:

+386,8 mm at 11,098055 sec, load step 192, substep 15

–135,1 mm at 11,679105 sec, load step 202 substep 26

Displacement range at the end:

386,8 mm –(–135,1 mm) = 521,9 mm

corresponding amplitude is 261 mm

Actual amplification is

386,8 mm / 232 mm = 1,69

Behaviour factor q is

q = 2273 mm / 386,8 mm = 5,9

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*232 mm = 464 mm

A driven run with base displacement amplitude 0,464 m and DAMP 0,023 over 30 secs goes up to +4539 mm / –4540 mm

V = 4540 mm / 464 mm = 9,8

or delta = 0,321 or D = 0,051

A plastic run over 15 seconds with 10 driven periods (16 min I/O) shows offset with steadily increasing head amplitudes.

Head displacements last peaks before decay:

–144,3 mm at 10,57511 sec, load step 183 substep 18

+453,4 mm at 11,15616 sec, load step 193, substep 20

Displacement range at the end:

453,4 mm –(–144,3 mm) = 597,7 mm

corresponding amplitude is 299 mm

Actual amplification is

453,4 mm / 464 mm = 0,98

Behaviour factor q is

q = 4540 mm / 453,4 mm = 10,0

5.17 Model 16

5.17.1 Properties

Like Model 15 with three storeys but V-braces in the first and second storey.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	50x2
2. storey columns	260x12
2. storey waler	260x12
2. storey diagonal	50x2
3. storey columns	170x12
3. storey waler	170x12

Base shear 95 kN produces max stress

206 MPa in the first storey diagonal

217 MPa in the second storey diagonal (possibly due to different frame stiffnesses)

234 MPa in the third storey column

Based on the max stresses in the third storey waler this is a range of 12 %.

Head displacement of 76,6 mm.

5.17.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

278 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 0,945$

$T = 1,06$

Natural frequency of the system is (damping 0,001, 20 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,250	72,5
0,277 (interpolated)	100
0,300	124,2

time [s]	displacement [mm]
19,800	98,2
19,804 (interpolated)	100
19,850	120,7

Length of period

T = (19,804 s – 0,277 s) / 20 = 0,9764 s.

A driven run with base displacement amplitude 0,153 m and DAMP 0,020 over 20 secs goes up to +1391 mm / –1391 mm

V = 1391 mm / 153 mm = 9,1

or delta = 0,346 or D = 0,055

A driven run with base displacement amplitude 0,153 m and DAMP 0,018 over 20 secs goes up to +1543 mm / –1544 mm

V = 1544 mm / 153 mm = 10,1

or delta = 0,311 or D = 0,050

5.17.3 Seismic Design

With an elastic limit top storey drift of 76,6 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 153 mm.

A plastic run over 12 seconds with 10 driven periods (13 min I/O) shows offset with steadily increasing head amplitudes.

Head displacements last peaks before decay:

+240,4 mm at 9,32462 sec, load step 192, substep 17

–100,6 mm at 9,81282 sec, load step 202 substep 14

Displacement range at the end:

240,4 mm –(–100,6 mm) = 341,0 mm

corresponding amplitude is 171 mm

Actual amplification is

240,4 mm / 153 mm = 1,57

Behaviour factor q is

q = 1544 mm / 240,4 mm = 6,4

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*153 mm = 306 mm

A driven run with base displacement amplitude 0,306 m and DAMP 0,018 over 20 secs goes up to +3087 mm / –3087 mm

V = 3087 mm / 306 mm = 10,1

or delta = 0,311 or D = 0,050

A plastic run over 12 seconds with 10 driven periods (13 min I/O) shows a considerable offset of the top storey within the first positive peak with steadily increasing head amplitudes. The stiffer base and second storey seemingly remain symmetric, following about the size of the driving amplitudes.

Head displacements last peaks before decay:

+503,0 mm at 9,37344 sec, load step 193, substep 15

+82,1 mm at 9,86164 sec, load step 203 substep 11

Displacement range at the end:

503 mm –(+82,1 mm) = 420,9 mm

corresponding amplitude is 210 mm

Actual amplification is

503,0 mm / 306 mm = 1,64

Behaviour factor q is

q = 3087 mm / 503 mm = 6,1

5.18 Model 17

5.18.1 Properties

Like Model 16 with three storeys but V-braces in all storeys.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	90x3
2. storey columns	260x12
2. storey waler	260x12
2. storey diagonal	80x3
3. storey columns	170x12
3. storey waler	170x12
3. storey diagonal	55x3

Base shear 245 kN produces max stress

232 MPa in the first storey diagonal

218 MPa in the second storey diagonal

204 MPa in the third storey diagonal

Based on the max stresses in the third storey waler this is a range of 12 %.

Head displacement of 26,6 mm.

5.18.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

43,0 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w_{,max})} = 2{,}40$

$T = 0{,}416$

Natural frequency of the system is (damping 0,001, 10 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,1040	75,4
0,112 (interpolated)	100
0,1248	136,2

time [s]	displacement [mm]
9,880	93,6
9,894 (interpolated)	100
9,9008	103,3

Length of period

T = (9,894 s – 0,112 s) / 26 = 0,3762 s.

A driven run with base displacement amplitude 0,100 m and DAMP 0,014 over 10 secs goes up to +538,9 mm / –538,9 mm

V = 539 mm / 100 mm = 5,4

or delta = 0,583 or D = 0,093

A driven run with base displacement amplitude 53,2 mm and DAMP 0,0075 over 10 secs goes up to +528,8 mm / –528,8 mm

V = 528,8 mm / 53,2 mm = 9,94

or delta = 0,316 or D = 0,050

5.18.3 Seismic Design

With an elastic limit top storey drift of 26,6 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 53,2 mm.

A plastic run over 5 seconds with 10 driven periods (13 min I/O) shows one major head peak and then offset with increasing amplitudes.
Head displacements: first positive peak and last peaks before decay:
+71,1 mm at 0,22572 sec, load step 13, substep 12
+48,1 mm at 3,61152 sec, load step 193, substep 10
–48,0 mm at 3,79962 sec, load step 203 substep 19

Displacement range at the end:
48,1 mm –(–48,0 mm) = 96,1 mm
corresponding amplitude is 48 mm

Actual amplification is
71,1 mm / 53,2 mm = 1,34

Behaviour factor q is
q = 529 mm / 71 mm = 7,5

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*53,2 mm = 106,4 mm

A driven run with base displacement amplitude 0,1064 m and DAMP 0,0075 over 10 secs goes up to
+1058 mm / –1058 mm
V = 1058 mm / 106,4 mm = 9,9
or delta = 0,316 or D = 0,050

A plastic run over 5 seconds with 10 driven periods (10 min I/O) shows a considerable offset of the top storey within the first positive peak with steadily increasing head amplitudes, which are much smaller than the drive. Middle storey is almost parallel to top storey. The bottom storey has much larger amplitudes, but still not as much as the drive.
Head displacement first peak:
+96,0 mm at 0,24453 sec, load step 14 substep 11
Displacement of bottom waler last peaks before decay

+84,1 mm at 3,51747 sec, load step 188, substep 19

–49,9 mm at 3,70557 sec, load step 198 substep 11

Displacement range at the end:

84,1 mm –(–49,9 mm) = 134 mm

corresponding amplitude is 67 mm

Actual amplification is

96,0 mm / 106,4 mm = 0,90

Behaviour factor q is

q = 1058 mm / 96,0 mm = 11,0

5.19 Model 18

5.19.1 Properties

Like Model 17 with three storeys but a frame in the first storey.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	–
2. storey columns	260x12
2. storey waler	260x12
2. storey diagonal	60x3
3. storey columns	170x12
3. storey waler	170x12
3. storey diagonal	55x1,5

Base shear 153 kN produces max stress

232 MPa in the first storey column

218-230 MPa in the second storey diagonal (along the length, no end bending effects)

230 MPa in the third storey diagonal, due to bending at the ends up to 242 MPa

Based on the max stresses in the third storey waler this is a range of 12 %.

Head displacement of 66,7 mm.

5.19.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

194 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 1,13$

$T = 0,884$

Natural frequency of the system is (damping 0,001, 20 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,2069	98,8
0,2081 (interpolated)	100
0,2257	118,1

time [s]	displacement [mm]
19,3179	99,3
19,3194 (interpolated)	100
19,3367	107,9

Length of period

$T = (19,319 \text{ s} - 0,208 \text{ s}) / 23 = 0,8309 \text{ s}.$

A driven run with base displacement amplitude 133,4 mm and DAMP 0,018 over 20 secs goes up to +1083 mm / –1083 mm

V = 1083 mm / 133,4 mm = 8,1

or delta = 0,387 or D = 0,062

A driven run with base displacement amplitude 133,4 mm and DAMP 0,018 over 20 secs goes up to +1083 mm / –1083 mm

V = 1083 mm / 133,4 mm = 8,1

or delta = 0,387 or D = 0,062

A driven run with base displacement amplitude 133,4 mm and DAMP 0,014 over 15 secs goes up to +1378 mm / –1378 mm

V = 1378 mm / 133,4 mm = 10,3

or delta = 0,304 or D = 0,0484

5.19.3 Seismic Design

With an elastic limit top storey drift of 66,7 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 133,4 mm.

A plastic run over 10 seconds with 10 driven periods (10 min I/O) shows one major head peak and then offset with increasing amplitudes.

Head displacements: first positive peak and last peaks before decay:

+204,9 mm at 0,456995 sec, load step 12, substep 11

+195,5 mm at 7,935095 sec, load step 192, substep 11

–78,9 mm at 8,350545 sec, load step 202 substep 18

Displacement range at the end:

195,5 mm –(–78,9 mm) = 274,4 mm

corresponding amplitude is 137 mm

Actual amplification is

204,9 mm / 133,4 mm = 1,54

Behaviour factor q is

q = 1378 mm / 205 mm = 6,7

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Assuming a behaviour factor of 4 for this configuration the driving amplitude should be

2*113,4 mm = 268,8 mm

A driven run with base displacement amplitude 268,8 mm and DAMP 0,014 over 15 secs goes up to +2756 mm / –2757 mm

V = 2757 mm / 268,8 mm = 10,3

or delta = 0,306 or D = 0,0487

A plastic run over 10 seconds with 10 driven periods (17 min I/O) shows one major head peak and then offset with increasing amplitudes, so that consecutive positive peaks are higher than the first one.

Head displacements: first positive peak and last peaks before decay:

+272,3 mm at 0,49854 sec, load step 13, substep 11

+288,9 mm at 7,97664 sec, load step 193, substep 26

–58,8 mm at 8,39209 sec, load step 203 substep 11

Displacement range at the end:

288,9 mm –(–58,8 mm) = 347,7 mm

corresponding amplitude is 174 mm

Actual amplification is

288,9 mm / 266,8 mm = 1,08

Behaviour factor q is

q = 2757 mm / 288,9 mm = 9,5

5.20 Model 19

5.20.1 Properties

Like Model 18 with three storeys but a frame in the first and second storey.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	–
2. storey columns	230x12
2. storey waler	230x12
2. storey diagonal	–
3. storey columns	170x12
3. storey waler	170x12
3. storey diagonal	55x1,5

Base shear 111 kN produces max stress
225 MPa in the first storey waler
225 MPa in the second storey column
211-233 MPa in the third storey diagonal, due to bending at the ends
Based on the max stresses in the third storey waler this is a range of 3 %.
Head displacement of 101,5 mm.

5.20.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:
386 mm yields app.
$f = 1/(2\pi) * \sqrt{(g / w_{,max})} = 0{,}80$
$T = 1{,}25$

Natural frequency of the system is (damping 0,001, 20 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,3120	99,4
0,3130 (interpolated)	100
0,3510	121,9

time [s]	displacement [mm]
18,7980	89,3
18,8216 (interpolated)	100
18,8370	107,0

Length of period

T = (18,822 s – 0,312 s) / 16 = 1,1569 s.

A driven run with base displacement amplitude 203 mm and DAMP 0,020 over 20 secs goes up to +2109 mm / –2110 mm

V = 2110 mm / 203 mm = 10,4

or delta = 0,302 or D = 0,048

A driven run with base displacement amplitude 203 mm and DAMP 0,021 over 20 secs goes up to +2013 mm / –2013 mm

V = 2110 mm / 203 mm = 9,92

or delta = 0,317 or D = 0,0504

5.20.3 Seismic Design

With an elastic limit top storey drift of 101,5 mm

and an assumed behaviour factor of 2

the base displacement amplitude should be 203 mm.

A plastic run over 15 seconds with 10 driven periods (11 min I/O) shows offset with increasing amplitudes.

Head displacements: last peaks before decay:

+361,3 mm at 11,048395 sec, load step 192, substep 14

–109,3 mm at 11,626845 sec, load step 202 substep 19

Displacement range at the end:

361,3 mm –(–109,3 mm) = 470,6 mm

corresponding amplitude is 235 mm

Actual amplification is

361,3 mm / 203 mm = 1,78

Behaviour factor q is

q = 2110 mm / 361 mm = 5,8

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*203 mm = 406 mm

A driven run with base displacement amplitude 406 mm and DAMP 0,021 over 20 secs goes up to +4025 mm / –4027 mm

V = 4027 mm / 406 mm = 9,92

or delta = 0,317 or D = 0,0504

A plastic run over 15 seconds with 10 driven periods (14 min I/O) shows S-shaped offset with increasing amplitudes.

Head displacements: last peaks before decay:

+383,9 mm at 11,048395 sec, load step 192, substep 14

–197,7 mm at 11,626845 sec, load step 202 substep 19

Displacement range at the end:

383,9 mm –(–197,7 mm) = 581,6 mm

corresponding amplitude is 291 mm

Actual amplification is

384 mm / 406 mm = 0,95

Behaviour factor q is

q = 4027 mm / 384 mm = 10,5

5.21 Model 20

5.21.1 Properties

Like Model 17 with three storeys but a frame in the middle.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	60x3
2. storey columns	260x12
2. storey waler	260x12
2. storey diagonal	80x3
3. storey columns	170x12
3. storey waler	170x12
3. storey diagonal	60x1,5

Base shear 138 kN produces max stress

217-226 MPa in the first storey diagonal, due to bending at the top end

232 MPa in the second storey column

224-242 MPa in the third storey diagonal, due to bending at the bottom end

Based on the max stresses in the third storey waler this is a range of 10 %.

Base shear is reduced to 134 kN as elastic limit load.

Head displacement of 53,3 mm.

5.21.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

155 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 1,29$

$T = 0,78$

Natural frequency of the system is (damping 0,001, 15 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,1750	67,9

0,1962 (interpolated)	100
0,2100	120,9

time [s]	displacement [mm]
14,210	85,9
14,237 (interpolated)	100
14,2450	104,4

Length of period

T = (14,237 s – 0,196 s) / 19 = 0,7390 s.

A driven run with base displacement amplitude 106,6 mm and DAMP 0,016 over 15 secs goes up to

+884 mm / –884 mm

V = 884 mm / 106,6 mm = 8,29

or delta = 0,379 or D = 0,060

A driven run with base displacement amplitude 106,6 mm and DAMP 0,0135 over 15 secs goes up to

+1045 mm / –1045 mm

V = 1045 mm / 106,6 mm = 9,8

or delta = 0,320 or D = 0,0510

5.21.3 Seismic Design

With an elastic limit top storey drift of 53,3 mm

and an assumed behaviour factor of 2

the base displacement amplitude should be 106,6 mm.

A plastic run over 10 seconds with 10 driven periods (17 min I/O) shows one major positive peak and then offset with increasing amplitudes.

Head displacements: first positive and last peaks before decay:

+140,3 mm at 0,40645 sec, load step 12, substep 27

+119,8 mm at 7,05745 sec, load step 192, substep 11

–107,1 mm at 7,42695 sec, load step 202 substep 12

Displacement range at the end:

119,8 mm –(–107,1 mm) = 226,9 mm

corresponding amplitude is 113 mm

Actual amplification is

140,3 mm / 106,6 mm = 1,32

Behaviour factor q is

q = 1045 mm / 140,3 mm = 7,4

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be

2*106,6 mm = 213 mm

A driven run with base displacement amplitude 213 mm and DAMP 0,0135 over 15 secs goes up to

+2048 mm / –2048 mm

V = 2048 mm / 213 mm = 9,62

or delta = 0,327 or D = 0,0520

A plastic run over 10 seconds with 10 driven periods (13 min I/O) shows one major positive peak and then offset with increasing amplitudes.

Head displacements: first positive and last peaks before decay:

+195,2 mm at 0,48035 sec, load step 14, substep 11

+184,6 mm at 7,05745 sec, load step 192, substep 12

–90,1 mm at 7,42695 sec, load step 202 substep 18

Displacement range at the end:

184,6 mm –(–90,1 mm) = 274,7 mm

corresponding amplitude is 137 mm

Actual amplification is

195,2 mm / 213 mm = 0,92

Behaviour factor q is

q = 2048 mm / 195,2 mm = 10,5

5.22 Model 21

5.22.1 Properties

Like Model 20 with three storeys but frames and Vs reversed.

Seismic design with respect to different storey loads:

1/6 in storey 1

1/3 in storey 2

1/2 in storey 3

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	–
2. storey columns	260x12
2. storey waler	260x12
2. storey diagonal	65x3
3. storey columns	210x12
3. storey waler	210x12
3. storey diagonal	–

Base shear 150 kN produces max stress

227 MPa in the first storey column

240 MPa in the second storey diagonal

232 MPa in the third storey column

Based on the max stresses in the second storey diagonal this is a range of 5 %.

Base shear is reduced to 146 kN as elastic limit load.

Head displacement of 103,3 mm.

5.22.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

279 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 1,29$

$T = 0,94$

Natural frequency of the system is (damping 0,001, 20 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,2700	79,6
0,2895 (interpolated)	100
0,3150	126,6

time [s]	displacement [mm]
19,080	81,6
19,112 (interpolated)	100
19,1250	107,8

Length of period

$T = (19,112 \text{ s} - 0,290 \text{ s}) / 20 = 0,9411 \text{ s}.$

A driven run with base displacement amplitude 206,6 mm and DAMP 0,011 over 20 secs goes up to +3633 mm / –3637 mm

$V = 3637 \text{ mm} / 206,6 \text{ mm} = 17,6$

or delta = 0,179 or D = 0,028

A driven run with base displacement amplitude 206,6 mm and DAMP 0,0195 over 15 secs goes up to

+2084 mm / –2085 mm

V = 2085 mm / 206,6 mm = 10,1

or delta = 0,311 or D = 0,050

5.22.3 Seismic Design

With an elastic limit top storey drift of 103,3 mm

and an assumed behaviour factor of 2

the base displacement amplitude should be 206,6 mm.

A plastic run over 12 seconds with 10 driven periods (12 min I/O) shows one major positive peak and then offset with increasing amplitudes.

Head displacements: first positive and last peaks before decay:

+314,7 mm at 0,56466 sec, load step 13, substep 12

+325,5 mm at 9,03456 sec, load step 193, substep 17

–165,8 mm at 9,50511 sec, load step 203 substep 15

Displacement range at the end:

325,5 mm –(–165,8 mm) = 491,3 mm

corresponding amplitude is 245 mm

Actual amplification is

325,5 mm / 206,6 mm = 1,58

Behaviour factor q is

q = 2085 mm / 325,5 mm = 6,4

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be

2*206,6 mm = 413 mm

A driven run with base displacement amplitude 413 mm and DAMP 0,0195 over 15 secs goes up to

+4170 mm / –4169 mm

V = 4170 mm / 413 mm = 10,1

or delta = 0,311 or D = 0,050

A plastic run over 12 seconds with 10 driven periods (14 min I/O) shows offset with increasing amplitudes.
Head displacements: last peaks before decay:
+487,6 mm at 9,03456 sec, load step 193, substep 20
–51,8 mm at 9,50511 sec, load step 203 substep 13

Displacement range at the end:
487,6 mm –(–51,8 mm) = 539,4 mm
corresponding amplitude is 270 mm

Actual amplification is
487,6 mm / 413 mm = 1,18

Behaviour factor q is
q = 4170 mm / 487,6 mm = 8,6

5.23 Model 10 amended

5.23.1 Properties

Seismic design with respect to different storey loads:
1/3 in storey 1
2/3 in storey 2

1. storey columns	400x12
1. storey waler	400x12
2. storey columns	200x12
2. storey waler	200x12.

At a total of 80 kN base shear the max stresses were
127 MPa at the first storey waler
134 MPa at the second storey column

Base shear was changed to 140 kN as elastic limit load.
Head displacement is 101,8 mm.

5.23.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

159,4 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 1{,}25$

$T = 0{,}80$

Natural frequency of the system is (damping 0,001):

time [s]	displacement [mm]
0,2400	80,8
0,257 (interpolated)	100
0,2800	126,0

time [s]	displacement [mm]
14,120	80,8
14,156 (interpolated)	100
14,160	102,3

Length of period

$T = (14{,}156\text{ s} - 0{,}257\text{ s}) / 15 = 0{,}9266\text{ s}$

A driven run with base displacement amplitude 203,6 mm and DAMP 0,016 over 15 secs goes up to +2209 mm / –2211 mm

$V = 2211\text{ mm} / 203{,}6\text{ mm} = 10{,}9$

or delta = 0,289 or D = 0,046

A driven run with base displacement amplitude 203,6 mm and DAMP 0,0175 over 15 secs goes up to

+2026 mm / –2027 mm

V = 2027 mm / 203,6 mm = 9,96

or delta = 0,316 or D = 0,050

5.23.3 Seismic Design

With an elastic limit top storey drift of 101,8 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 203,6 mm.

A plastic run over 12 seconds with 10 driven periods (9 min I/O) shows offset with increasing amplitudes.

Head displacements: last peaks before decay:
+378,8 mm at 8,84903 sec, load step 192, substep 15
–98,8 mm at 9,31233 sec, load step 202 substep 15

Displacement range at the end:
378,8 mm –(–98,8 mm) = 477,6 mm
corresponding amplitude is 239 mm

Actual amplification is
378,8 mm / 203,6 mm = 1,86

Behaviour factor q is
q = 2027 mm / 378,8 mm = 5,4

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be
2*203,6 mm = 407 mm

A driven run with base displacement amplitude 407 mm and DAMP 0,0175 over 15 secs goes up to
+4053 mm / –4055 mm

V = 4055 mm / 407 mm = 9,96

or delta = 0,316 or D = 0,050

A plastic run over 12 seconds with 10 driven periods (12 min I/O) shows one major peak and then offset with increasing amplitudes.

Head displacements: first positive and last peaks before decay:

+295,5 mm at 0,50963 sec, load step 12, substep 15

+265,1 mm at 8,84903 sec, load step 192, substep 15

–298,2 mm at 9,31233 sec, load step 202 substep 15

Displacement range at the end:

265,1 mm –(–298,8 mm) = 563,9 mm

corresponding amplitude is 282 mm

Actual amplification is

298,2 mm / 407 mm = 0,73

Behaviour factor q is

q = 4055 mm / 298,2 mm = 13,6

5.24 eModel 22

5.24.1 Properties

Single storey frame.

1. storey columns	400x12
1. storey waler	400x12

Base shear 322 kN produces max stress

234 MPa in the first storey frame corners

Head displacement is 52,3 mm.

5.24.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

26,0 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 3,09$

$T = 0,3235$

Natural frequency of the system is (damping 0,001, 20 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,11323	89,3
0,121 (interpolated)	100
0,12940	111,2

time [s]	displacement [mm]
6,9229	90,8
6,937 (interpolated)	100
6,9391	101,6

Length of period

T = (6,937 s – 0,121 s) / 15 = 0,4544 s

A driven run with base displacement amplitude 100 mm and DAMP 0,0075 over 7 secs goes up to +937 mm / –983 mm

V = 938 mm / 100 mm = 9,4

or delta = 0,335 or D = 0,053

A driven run with base displacement amplitude 100 mm and DAMP 0,0070 over 12 secs goes up to +1009 mm / –1009 mm

V = 1009 mm / 100 mm = 10,1

or delta = 0,311 or D = 0,050

5.24.3 Seismic Design

With an elastic limit top storey drift of 52,3 mm

and an assumed behaviour factor of 2

the base displacement amplitude should be 104,6 mm.

A plastic run over 7 seconds with 10 driven periods (7 min I/O) shows one major positive peak, then offset with increasing amplitudes.

Head displacements: first positive and last peaks before decay:

+153,9 mm at 0,24992 sec, load step 12 substep 42

+161,7 mm at 4,33952 sec, load step 192, substep 19

–51,2 mm at 4,56672 sec, load step 202 substep 26

Displacement range at the end:

161,7 mm –(–51,2 mm) = 212,9 mm

corresponding amplitude is 106 mm

Actual amplification is

161,7 mm / 104,6 mm = 1,55

Behaviour factor q is

q = 1009 mm / 161,7 mm = 6,2

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be

2*104,6 mm = 209 mm

A driven run with base displacement amplitude 209 mm and DAMP 0,0070 over 12 secs goes up to

+2111 mm / –2111 mm

V = 2111 mm / 209 mm = 10,1

or delta = 0,311 or D = 0,050

A plastic run over 7 seconds with 10 driven periods (8 min I/O) shows offset with increasing amplitudes.

Head displacements: last peaks before decay:

+267,0 mm at 4,33952 sec, load step 192, substep 33

+9,7 mm at 4,56672 sec, load step 202 substep 29

267,0 mm –(+9,7 mm) = 257,3 mm

corresponding amplitude is 129 mm

Actual amplification is

267 mm / 209 mm = 1,30

Behaviour factor q is

q = 2111 mm / 267 mm = 7,9

5.25 Model 23

5.25.1 Properties

Re-running five storey frame under different conditions.

Seismic design with respect to different storey loads:

1/15 in storey 1

2/15 in storey 2

3/15 in storey 3

4/15 in storey 4

5/15 in storey 5

1. storey columns and waler	400x12
2. storey columns and waler	310x12
3. storey columns and waler	270x12
4. storey columns and waler	230x12
5. storey columns and waler	150x12

At a total of 120 kN base shear the max stresses were

app. 262 MPa in the first storey waler

app. 244 MPa in the second storey waler

app. 262 MPa in the third storey waler

app. 253 MPa in the fourth storey waler

app. 244 MPa in the fifth storey column

Base shear was changed to 107 kN as elastic limit load.

Head displacement is 261 mm.

5.25.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

1457 mm yields app.

$f = 1/(2\pi) * \sqrt{(g / w,max)} = 0,41$

$T = 2,4$

Natural frequency of the system is (damping 0,001, 50 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,60	59,1
0,652 (interpolated)	100
0,70	137,5

time [s]	displacement [mm]
48,20	87,3
48,233 (interpolated)	100
48,30	125,6

Length of period

$T = (48,233 \text{ s} - 0,652 \text{ s}) / 23 = 2,069 \text{ s}$

A driven run with base displacement amplitude 522 mm and DAMP 0,020 over 35 secs goes up to +11661 mm / –11611 mm

$V = 11661 \text{ mm} / 522 \text{ mm} = 22,3$

or delta = 0,141 or D = 0,022

A driven run with base displacement amplitude 522 mm and DAMP 0,045 over 35 secs goes up to +5477 mm / –5476 mm

V = 5477 mm / 522 mm = 10,5

or delta = 0,299 or D = 0,048

A driven run with base displacement amplitude 522 mm and DAMP 0,047 over 35 secs goes up to +5247 mm / –5246 mm

V = 5247 mm / 522 mm = 10,05

or delta = 0,313 or D = 0,050

5.25.3 Seismic Design

With an elastic limit top storey drift of 261 mm

and an assumed behaviour factor of 2

the base displacement amplitude should be 522 mm.

A plastic run over 25 seconds with 10 driven periods (14 min I/O) shows offset with increasing amplitudes.

Head displacements: last peaks before decay:

+889,0 mm at 19,86240 sec, load step 193, substep 16

–389,3 mm at 20,89690 sec, load step 203 substep 16

Displacement range at the end:

889,0 mm –(–389,3 mm) = 1278,3 mm

corresponding amplitude is 639 mm

Actual amplification is

889,0 mm / 522 mm = 1,70

Behaviour factor q is

q = 5247 mm / 889,0 mm = 5,9

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*522 mm = 1044 mm

A driven run with base displacement amplitude 1044 mm and DAMP 0,047 over 35 secs goes up to +10495 mm / –10493 mm

V = 10495 mm / 1044 mm = 10,05

or delta = 0,313 or D = 0,050

A plastic run over 25 seconds with 10 driven periods (20 min I/O) offset with increasing amplitudes.

Head displacements: last peaks before decay:

+1266 mm at 19,86240 sec, load step 193, substep 14

–97,1 mm at 20,89690 sec, load step 203 substep 21

Displacement range at the end:

1266 mm –(–97,1 mm) = 1363 mm

corresponding amplitude is 682 mm

Actual amplification is

1266 mm / 1044 mm = 1,21

Behaviour factor q is

q = 10495 mm / 1266 mm = 8,3

5.26 Model 24

5.26.1 Properties

Single storey frame with V-bracing.

1. storey columns	400x12
1. storey waler	400x12
1. storey diagonal	90x3

Base shear 250 kN produces max stress

235 MPa in the first storey diagonals

Head displacement is 8,97 mm.

5.26.2 Dynamics

Rayleigh-Morleigh estimation (160 kN horizontal per storey) of the natural frequency:

5,74 mm yields app.

f = 1/(2π) * √(g / w,max) = 6,58

T = 0,152

Natural frequency of the system is (damping 0,001, 3 secs, step base displacement 100 mm)

time [s]	displacement [mm]
0,03750	84,9
0,0414 (interpolated)	100
0,0450	114,0

time [s]	displacement [mm]
2,93250	97,7
2,9399 (interpolated)	100
2,940	100,04

Length of period

T = (2,9399 s – 0,0414 s) / 19 = 0,15255 s

A driven run with base displacement amplitude 17,9 mm and DAMP 0,001 over 4,5 secs goes up to +401,0 mm / –401,6 mm

V = 401 mm / 17,9 mm = 22,3

or delta = 0,140 or D = 0,022

A driven run with base displacement amplitude 17,9 mm and DAMP 0,00225 over 4,5 secs goes up to

+186,1 mm / –186,1 mm

V = 186,1 mm / 17,9 mm = 10,4

or delta = 0,302 or D = 0,048

A driven run with base displacement amplitude 17,9 mm and DAMP 0,00235 over 4,5 secs goes up to

+178,3 mm / –178,3 mm

V = 178,3 mm / 17,9 mm = 9,96

or delta = 0,315 or D = 0,050

5.26.3 Seismic Design

With an elastic limit top storey drift of 8,97 mm
and an assumed behaviour factor of 2
the base displacement amplitude should be 17,9 mm.

A plastic run over 2 seconds with 10 driven periods (5 min I/O) shows one major peak and then increasing amplitudes without offset.

Head displacements: first positive peak and last peaks before decay:
+24,6 mm at 0,0839025 sec, load step 12, substep 16
+14,5 mm at 1,4568525 sec, load step 192, substep 10
–14,6 mm at 1,5331275 sec, load step 202 substep 11

Displacement range at the end:
14,5 mm –(–14,6 mm) = 29,1 mm
corresponding amplitude is 14,6 mm

Actual amplification is
24,6 mm / 17,9 mm = 1,37

Behaviour factor q is
q = 178,3 mm / 24,6 mm = 7,2

Assuming a behaviour factor of 4 for this configuration the driving amplitude should be 2*17,9 mm = 35,8 mm

A driven run with base displacement amplitude 35,8 mm and DAMP 0,00235 over 4,5 secs goes up to

+356,6 mm / –356,6 mm

V = 356,6 mm / 35,8 mm = 9,96

or delta = 0,315 or D = 0,050

A plastic run over 2 seconds with 10 driven periods (6 min I/O) shows one major peak and then increasing amplitudes without offset.

Head displacements: first positive peak and last peaks before decay:

+33,1 mm at 0,0839025 sec, load step 12, substep 12

+19,6 mm at 1,464480 sec, load step 193, substep 20

–17,4 mm at 1,540755 sec, load step 203 substep 10

Displacement range at the end:

19,6 mm –(–17,4 mm) = 37,0 mm

corresponding amplitude is 18,5 mm

Actual amplification is

33,1 mm / 35,8 mm = 0,92

Behaviour factor q is

q = 356,6 mm / 33,1 mm = 10,8

6 On Damping

With a damping parameter “Lehrsches Dämpfungsmaß” of 0,05 or 5 % the relation of adjacent amplitudes should be as follows (“Mensinger-Formel”):

k = e^(ANZ * delta)

k = e^(1 * 0,05) = 1,051

k = e^(2 * 0,05) = 1,105

k = e^(5 * 0,05) = 1,284

k = e^(10 * 0,05) = 1,649

Upside down:

delta = 1/ANZ * ln [Amp(N) / Amp (N+ANZ)

delta = 1/15 * ln [1 / 0,45] = 0,053

There seems to be a mismatch between a decaying response and a driven response for the same damping. E.g. Model 18:

Decaying response with DAMP 0,001: k(23) = 1,95

delta = 1/23 * ln(1,95) = 0,0290

D = 0,0290 / 2π = 0,0046

Driven response with DAMP 0,018 – damping should be 18 times larger, but is only

delta = 0,387 / D = 0,062, which is only 13,3 times larger.

7 Plots

The displacement plots ux were done with a scaling factor of 20.

8 Results

- Stiffer structures need a smaller parameter DAMP to reach the same limit amplitude when driven in their natural frequency.

- Gravity and geometrical nonlinearity (second order effects) were not activated, i.e. stability problems (compare ”shakedown”) of the configurations were not investigated.

- There seems to be a change in periods for the plastic runs. Refer to the data of time-history displacements. Check if the end-peaks of the elastic driven runs occur at the same time of the plastic ones.

9 References

[1] Çeltikçi, N.T., Çeltikçi, Y., Güçlü, S.: A New Method for Calculation of X-Type Bracings, pp 187-196 in: Yardımcı, N., Aydöner, B., Gür'eş, Y., Yorgun, C. (eds.): Proceedings of the International Symposium “Steel Structures: Culture & Sustainability 2010” (ECCS/TUCSA), 21-23 September 2010, Istanbul, Turkey.

[2] Knoedel, P.: On the Dynamics of Steel Structures with X-Type Bracing. Stahlbau 80 (2011), No. 8, p. 566-571.

[3] Knoedel, P., Hrabowski, J.: Seismic Design in Plant Construction – Shortcomings of EC8. In: Dunai, L., Ivany, M., Jarmai, K., Kovacs, N., Vigh, L.G. (eds.): Proceedings Vol. B, p.

1083–1988, Eurosteel 2011, 6th European Conference on Steel and Composite Structures, Budapest, Hungary, 31.08.-02.09.2011.

[4] Knoedel, P., Hrabowski, J.: Yield Limit vs. Behaviour Factor in Seismic Design. Proceedings, NSCC 2012 Nordic Steel Construction Conference, 5-7 September 2012, Oslo, Norway, pp 147-155.

[5] Schmitt, A.: Auswanderungserscheinungen eines parametererregten Rahmens aus elastisch-plastischem Werkstoff. Dissertation, Universität Karlsruhe 1986. Berichte der Versuchsanstalt für Stahl, Holz und Steine der Universität Fridericiana in Karlsruhe, 4. Folge – Heft 15.